6.7 Cvičenia

1. Nájdite extremály pre dané funkcionály:

a) $v[x(t)] = \int_{t_0}^{t_1} x'(1 + t^2 x')dt$; [[$x = C_1 t^{-1} + C_2$]]

b) $v[x(t)] = \int_{t_0}^{t_1} (tx' + x'^2)dt$; [[$x = C_1 t + C_2 - 0{,}25t^2$]]

c) $v[x(t)] = \int_{t_0}^{t_1} (16x^2 - x'' + t^2)dt$; [[$x = C_1 e^{2t} + C_2 e^{-2t} + C_3 \cos 2t + C_4 \sin 2t$]]

d) $v[x(t)] = \int_{t_0}^{t_1} (2tx + (x''')^2)dt$; [[$x = \frac{1}{7!}t^7 + C_1 t^5 + C_2 t^4 + C_3 t^3 + C_4 t^2 + C_5 t + C_6$]]

e) $v[x(t), y(t)] = \int_{t_0}^{t_1} (2xy - 2x^2 + x'^2 - y'^2)dt$;

[[$x = (C_1 t + C_2)\cos t + (C_3 t + C_4)\sin t; y = 2x + x''$]]

f) $v[x(t)] = \int_{t_0}^{t_1} (x''^2 - 2x'^2 + x^2 - 2x\sin t)dt$;

[[$x = (C_1 t + C_2)\cos t + (C_3 t + C_4)\sin t - 0{,}25t^2 \sin t$]]

g) $v[x(t)] = \int_{t_0}^{t_1} (x'''^2 + x^2 - 2xt^3)dt$.

[[$x = C_1 e^t + C_2 e^{-t} + e^{0,5t}(C_3 \cos\frac{\sqrt{3}}{2}t + C_4 \sin\frac{\sqrt{3}}{2}t) + e^{-0,5t}(C_5 \cos\frac{\sqrt{3}}{2}t + C_6 \sin\frac{\sqrt{3}}{2}t) + t^3$]]

2. Nájdite extremály pre dané funkcionály a dané okrajové podmienky:

a) $v[x(t)] = \int_0^1 (x'^2 - 12tx)dt;\quad x(0) = 1, x(1) = 0$; [[$x = 1 - t^3$]]

b) $v[x(t)] = \int_0^1 (12tx + xx' + x'^2)dt;\quad x(0) = 1, x(1) = 4$; [[$x = 1 + 2t + t^3$]]

c) $v[x(t)] = \int_0^2 \sqrt{x(1 + x'^2)}dt; x(0) = 2, x(2) = 1$; [[$x = 1 + 0{,}25(t-2)^2$]]

d) $v[x(t)] = \int_0^{t_1} (x'^4 - x'^2)dt; \quad x(0) = 0, x(t_1) = x_1;$ [[$x = 0{,}2 + 1{,}25(t - 1{,}2)^2$]]

e) $v[x(t), y(t)] = \int_0^1 (x'^2 + y'^2 + x'y')dt$;
$x(0) = -1, x(1) = 1, y(0) = 2, y(1) = -2$ [[$x = 2t - 1; y = -4t + 2$]]

f) $v[x(t), y(t)] = \int_0^1 (x'^2 + y'^2 + 2y)dt$;
$x(0) = 0, x(1) = 1, y(0) = 1, y(1) = 3/2$ [[$x = t; y = 0{,}5t^2 + 1$]]

g) $v[x(t), y(t)] = \int_{-1}^1 (\frac{1}{3}x'^3 - y'^2 + 2ty)dt$.
$x(-1) = -1, x(1) = 1, y(-1) = 2, y(1) = 0$ [[$x = t; y = -\frac{1}{6}(t^3 + 5t - 6)$]]

3. Nájdite extremály pre dané funkcionály a dané okrajové podmienky:

a) $v[x(t)] = \int_{-1}^2 x'(1 + t^2 x')dt; \quad x(-1) = 1, x(2) = 4;$ [[Nemá extrémy;]]

b) $v[x(t)] = \int_1^2 x'(1 + t^2 x')dt; \quad x(1) = 3, x(2) = 5;$ [[Pre $x = 7 - 4/t$ - silný minimum;]]

c) $v[x(t)] = \int_1^2 (12x^2 + t^2 x'^2)dt; \quad x(1) = 1, x(2) = 8;$ [[Pre $x = t^3$ - silné maximum;]]

d) $v[x(t)] = \int_0^{\pi/4} (x^2 - x'^2 + 6x \sin 2t)dt; \quad x(0) = 0, x(\pi/4) = 1;$

[[Pre $x = \sin 2t$ - silné maximum;]]

e) $v[x(t)] = \int_1^3 (x'^2 + 12tx)dt; \quad x(1) = 0, x(3) = 26;$ [[Pre $x = t^3 - 1$ - silné maximum;]]

f) $v[x(t)] = \int_0^2 (x^2 + x'^2 - 2tx)dt; \quad x(0) = 0, x(2) = 3.$

[[Pre $x = (\sinh t)(\sinh 2)^{-1} + t$ - silné minimum.]]