4.14 Cvičenia

1. Riešte diferenčné rovnice:

   a) $x(n+2)-6x(n+1)+5x(n)=0$, [[$x(n)=C_1+C_2 5^n$]]

   b) $x(n+3)-6x(n+2)+12x(n+1)-8x(n)=0$, [[$x(n)=2^n(C_1+C_2 n+C_3 n^2)$]]

   c) $x(n+2)+6x(n+1)+8x(n)=0$, [[$x(n)=C_1(-2)^n+C_2(-4)^n$]]

   d) $x(n+2)-2\sqrt{3}x(n+1)+4x(n)=0$,

   [[$x(n)=C_1(-2)^n+C_2(-4)^n$ $x(n)=2^n(C_1\cos(\pi n/6)+C_2\sin(\pi n/6))$]]

   e) $x(n+3)-7x(n+2)+16x(n+1)-12x(n)=0$,

   [[$x(n)=2^n(C_1+C_2 n)+C_3 3^n$]]

   f) $x(n+4)-4x(n+3)+12x(n+2)-16x(n+1)+16x(n)=0$,

   [[$x(n)=2^n\{(C_1+C_2 n)\cos(\pi n/3)+(C_3+C_4 n)\sin(\pi n/3))\}$]]

   g) $x(n+5)-x(n+4)-x(n+1)+x(n)=0.$

   [[$x(n)=C_1+C_2 n+C_3(-1)^n+C_4\cos(\pi n/2)+C_5\sin(\pi n/2)$]]

2. Riešte diferenčné rovnice:

   a) $2x(n+2)-3x(n+1)-5x(n)=2$, [[$x(n)=C_1(-1)^n+C_2\left(5/2\right)^n-1/3$]]

   b) $x(n+2)-5x(n+1)+6x(n)=1+n$, [[$x(n)=C_1 2^n+C_2 3^n+(n+5/2)/2$]]

   c) $x(n+2)-4x(n+1)+x(n)=n(n-1)$,

   [[$x(n)=C_1(2+\sqrt{3})^n+C_2(2-\sqrt{3})^n-(n^2-3n+2)/2$]]

   d) $x(n+2)-x(n+1)-6x(n)=5.3^n$, [[$x(n)=C_1 3^n+C_2(-2)^n+n3^{n-1}$]]

   e) $x(n+2)+x(n)=3^n(n+1)$,

   [[$x(n)=C_1\cos(\pi n/2)+C_2\sin(\pi n/2)+(n/10-2/25)3^n$]]

   f) $x(n+2)-x(n)=n\cos(\pi n/2)$,

   [[$x(n)=C_1+C_2(-1)^n+0{,}5(1-n)\cos(\pi n/2)$]]

   g) $x(n+2)-5x(n+1)+4x(n)=4^n-n^2$.

   [[$x(n)=C_1+C_2 4^n+4^n n/12+7n/54-n^2/18+n^3/9$]]

3. Určíte riešenie danej nehomogénnej diferenčnej rovnice vyhovujúce daným začiatočným podmienkam.

   a) $x(n+2)+4x(n+1)+3x(n)=5, x(0)=1, x(1)=-1$; [[$x(n)=5[(-3)^n+1]/8-(-1)^n/4$]]

   b) $\Delta^2 x(n)=16, x(0)=2, x(1)=3$; [[$x(n)=2-7n+8n^2$]]

   c) $x(n+3)-x(n+2)-x(n+1)+x(n)=n^2, x(0)=x(1)=x(2)=0$;

   [[$x(n)=n(n-1)(n-2)(n-3)/24$]]

   d) $x(n+2)-6x(n+1)+9x(n)=n3^n, x(0)=x(1)=0$;

   [[$x(n)=3^{n-3}n(n-1)(n-2)/2$]]

   e) $2x(n+2)-5x(n+1)+2x(n)=\cos(\pi n/3), x(0)=x(1)=0$;

   [[$x(n)=\left[2^n-\cos(\pi n/3)-\sqrt{3}\sin(\pi n/3)\right]/6$]]

   f) $x(n+2)+x(n)=1-(-1)^n, x(0)=0, x(1)=1$; [[$x(n)=\left[1-(-1)^n\right]/2$]]

   g) $x(n+2)-10x(n+1)+25x(n)=5^n-2n, x(0)=0, x(1)=-1/40$.

   [[$x(n)=\left[n^2/50-n/20+1/16\right]5^n-n/8-1/16$]]

4. Nájdite všeobecné riešenie daného systému homogénnych diferenčných rovníc.

a) $x(n+1)=3x(n)-2y(n), y(n+1)=2x(n)-y(n)$;

[[$x(n)=C_1+C_2(2n+1), y(n)=C_1+2nC_2$]]

b) $x(n+1)=-3x(n)-y(n), y(n+1)=x(n)-y(n)$;

[[$x(n)=(-2^n)(C_1+C_2n), y(n)=(-2^n)(-C_1+C_2(2-n))$]]

c) $x(n+1)=x(n)-y(n), y(n+1)=5x(n)+3y(n)$;

[[$\begin{aligned} x(n)&=8^{n/2}\left[-C_1\cos(\pi n/4)-C_2\sin(\pi n/4)\right],\\ y(n)&=8^{n/2}\left[(C_1+2C_2)\cos(\pi n/4)+(-2C_1+C_2)\sin(\pi n/4)\right]\end{aligned}$]]

d) $\begin{aligned} &x(n+1)=y(n)+z(n)/3, y(n+1)=x(n)+2y(n)-z(n),\\ &z(n+1)=3x(n)-z(n)\end{aligned}$;

[[$\begin{aligned} x(n)&=2C_1+3C_2\,2^n-C_3(-2)^n,\\ y(n)&=C_1+5C_2\,2^n+C_3(-2)^n,\\ z(n)&=3C_1+3C_2\,2^n+3C_3(-2)^n\end{aligned}$]]

e) $\begin{aligned} &x(n+1)=x(n)-3y(n), y(n+1)=2x(n)+4z(n),\\ &z(n+1)=-x(n)/3+y(n)+2z(n)\end{aligned}$.

[[$\begin{aligned} x(n)&=6C_1 3^n+(6C_2\cos(\pi n/2)+6C_3\sin(\pi n/2))2^n,\\ y(n)&=-4C_1 3^n+[(2C_3-4C_2)\cos(\pi n/2)+(2C_2+4C_3)\sin(\pi n/2)]2^n,\\ z(n)&=-6C_1 3^n+[(C_2-C_3)\cos(\pi n/2)-(C_2+C_3)\sin(\pi n/2)]2^n\end{aligned}$]]

5. Nájdite riešenie daného systému diferenčných rovníc vyhovujúce daným začiatočným podmienkam.

a) $x(n+1)=-x(n)+y(n), y(n+1)=2y(n), x(0)=1, y(0)=2$;

[[$x(n)=(2^{n+1}+(-1)^n)/3, y(n)=2^{n+1}$]]

b) $x(n+1)=2x(n)-y(n), y(n+1)=4y(n), x(0)=1, y(0)=2$;

[[$x(n)=2^{n+1}-4^n, y(n)=2.4^n$]]

c) $x(n+1)=3x(n)+y(n), y(n+1)=-5x(n)-y(n), x(0)=1, y(0)=1$;

[[$\begin{aligned} x(n)&=2^{n/2}(\cos(\pi n/4)+3\sin(\pi n/4)),\\ y(n)&=2^{n/2}(\cos(\pi n/4)-7\sin(\pi n/4))\end{aligned}$]]

d) $\begin{aligned} &x(n+1)=-x(n)+y(n)+z(n), y(n+1)=x(n)-y(n)+z(n),\\ &z(n+1)=x(n)+y(n)-z(n), x(0)=1, y(0)=z(0)=0\end{aligned}$

[[$x(n)=(1-(-2)^{n+1})/3, y(n)=z(n)=(1-(-2)^n)/3$]]

e) $\begin{aligned} &x(n+1)=y(n)+z(n), y(n+1)=x(n)+z(n), z(n+1)=x(n)+y(n),\\ &x(0)=-1, y(0)=1, z(0)=0\end{aligned}$;

[[$x(n)=(-1)^{n+1}, y(n)=(-1)^n, z(n)=0$]]

f) $x(n+1)=2x(n)+3y(n)+1, y(n+1)=x(n)+4y(n), x(0)=0, y(0)=-1$;

[[$x(n)=11/16+3n/4-11.5^n/16, y(n)=-5/16-n/4-11.5^n/16$]]

g) $x(n+1)=2x(n)+y(n)+2, y(n+1)=x(n)+2y(n)+3, x(0)=1, y(0)=1$;

[[$x(n)=(5.3^n-2n-5)/4, y(n)=(5.3^n+2n-5)/4$]]