3.7 Cvičenia

1. Nájdite riešenie systému diferenciálnych rovníc:
   a) $x' = 7x + 6y, y' = 2x + 6y$; [[$x = 2C_1e^{10t} + 3C_2e^{3t}, y = C_1e^{10t} - 2C_2e^{3t}$]]
   b) $x' = x - 5y, y' = 2x - y$; [[$x = 5C_1\cos 3t + 5C_2\sin 3t,\ y = C_1(\cos 3t) + 3\sin 3t) + C_2(\sin 3t - 3\cos 3t)$]]
   c) $x' = x - y, y' = -x + y$; [[$x = C_1 + C_2e^{2t}, y = C_1 - C_2e^{2t}$]]
   d) $x' = -y + z, y' = z, z' = -x + z$; [[$x = (C_2 + C_3)\cos t + (C_3 - C_2)\sin t, y = C_1e^t + C_2\cos t + C_3\sin t,\ z = C_1e^t - C_2\cos t + C_3\sin t$]]
   e) $x' = 4x + y + z, y' = 2x + 4y + z, z' = y + 4z$. [[$x = 3C_1e^{6t} + C_2e^{3t}, y = 4C_1e^{6t} + (C_2t - C_2 + C_3)e^{3t},\ z = 2C_1e^{6t} - (C_2t + C_3)e^{3t}$]]

2. Nájdite riešenie systému diferenciálnych rovníc:
   a) $x' = -2x + y - e^{2t}, y' = -3x + 2y + 6e^{2t}$; [[$x = C_1e^{-t} + C_2e^t + 2e^{2t}, y = C_1e^{-t} + 3C_2e^t + 9e^{2t}$]]
   b) $x' = -y + \mathrm{tg}\, t, y' = x + \mathrm{tg}^2 t$; [[$x = -C_1\sin t + C_2\cos t + 2, y = C_1\cos t + C_2t + \mathrm{tg}\, t$]]
   c) $x' = 2x + y, y' = x + 2y - 3e^{4t}$; [[$x = C_1e^t + C_2e^{3t} - e^{4t}, y = -C_1e^t + C_2e^{3t} - 2e^{4t}$]]
   d) $x' = 2x + 4y + \cos t, y' = -x - 2y + \sin t$; [[$x = -2C_1 - C_2(2t + 1) - 2\cos t - 3\sin t, y = C_1 + C_2t + 2\sin t$]]
   e) $x' = 2x + y - 2z - t, y' = 1 - x, z' = x + y - z + 1 - t$; [[$x = C_1e^t + C_2\cos t + C_3\sin t, y = -C_1e^t + C_3\cos t - C_2\sin t + t,\ z = C_2\cos t + C_3\sin t + 1$]]
   f) $x' = 2x, y' = 3x - 2y, z' = 2y + 3z + e^{3t}$. [[$x = 4C_1e^{2t}, y = 3C_1e^{2t} + 5C_2e^{-2t}, z = -6C_1e^{2t} - 2C_2e^{-2t} + C_3e^{3t} + te^{3t}$]]

3. Nájdite riešenie systému diferenciálnych rovníc s danými začiatočnými podmienkami
   a) $x' = -y, y' = -x, x(0) = 1, y(0) = -1$; [[$x = e^t, y = -e^t$]]
   b) $x' = -y, y' = 2x + 2y, x(0) = 1, y(0) = 1$; [[$x = e^t(\cos t - 2\sin t), y = e^t(\cos t + 3\sin t)$]]
   c) $x' = -y - z, y' = -x - z, z' = -x - y, x(0) = -1, y(0) = 0, z(0) = 1$; [[$x = -e^t, y = 0, z = e^t$]]
   d) $x' = -2x - 2y - 4z, y' = -2x + y - 2z, z' = 5x + 2y - 2z$; $x(0) = y(0) = z(0) = 1$; [[$x = 6e^t - e^{2t} - 4e^{3t}, y = 3e^t - 2e^{3t}, z = -6e^t + e^{2t} + 6e^{3t}$]]
   e) $x' = -2x - y + \sin t, y' = 4x + 2y + \cos t, x(\pi) = 1, y(\pi) = 2$; [[$x = 1 - 2(t - \pi) + 2\sin t, y = 4(t - \pi) - 2\cos t - 3\sin t$]]
   f) $x' = -y + z, y' = z + e^{-t}, z' = -x + z, x(0) = 0, y(0) = 1/2, z(0) = 0$. [[$x = 0{,}5(\cos t - 2\sin t - e^{-t}), y = (2e^t - 3e^{-t} + 3\cos t - \sin t)/4,\ z = (2e^t - e^{-t} - \cos t - 3\sin t)/4$]]