RzM_6b

streda 4. novembra 2026 7:35

PRIEBEH FUNKCIE:

① DEFINIČNÝ OBOR

② LIMITY VO VZDIALENÝCH BODOCH ($\pm\infty$): $\lim_{x\to\infty} f(x)$ resp. $\lim_{x\to-\infty} f(x)$

③ ASY (JEDNOSTRANNÉ LIMITY V BODOCH NESPOJITOSTI)

ASS: $y = kx + q$, kde $k = \lim_{x\to\pm\infty} \frac{f(x)}{x}$

$q = \lim_{x\to\pm\infty} (f(x) - kx)$

④ PÁRNOSŤ, NEPÁRNOSŤ

a) $\forall x \in D(f)$: ak $x \in D(f)$ tak aj $-x \in D(f)$

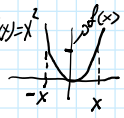
(SYMETRIA D(f) VZHĽADOM NA 0)

b) ak $f(-x) = f(x)$ - fcia je PÁRNA

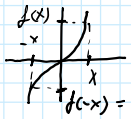
ak $f(-x) = -f(x)$ - fcia je NEPÁRNA

Inak, fcia je ANI PÁRNA ANI NEPÁRNA

Pr: $f(x) = x^2$, $f(-x) = f(x)$



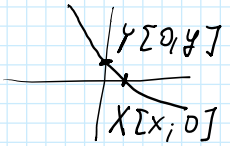
Pr: $f(x) = x^3$



⑤ PRIESEČNÍKY S OSAMI:

O_x: $X[x, 0] \to f(x) = 0$

O_y: $Y[0, y] \to f(0) = y$



⑥ MONOTÓNNOSŤ a LOKÁLNE EXTRÉMY

ak $f'(x) > 0$ fcia je RASTÚCA

ak $f'(x) < 0$ fcia je KLESAJÚCA

SB: $f'(x) = 0$ v $f'(x)$ nemení (SB $\in D(f)$)

⑦ KONVEXNOSŤ, KONKÁVNOSŤ, INFLEXNÉ BODY

$f'' > 0$ fcia je KONVEXNÁ $\cup$

$f'' < 0$ fcia je KONKÁVNA $\cap$

⑧ GRAF

Př: $f(x) = \frac{x^2}{x^2-1}$ ① $D(f) = \mathbb{R} - \{\pm 1\}$

② $\lim_{x\to\infty} \frac{x^2}{x^2-1} = 1$ $\lim_{x\to-\infty} \frac{x^{②}}{x^{②}-1} = 1$

③ ABS: $\lim_{x\to-1^-} \frac{x^2}{x^2-1} = \frac{1}{0^+} = \infty$ $\lim_{x\to-1^+} \frac{x^2}{x^2-1} = \frac{1}{0^-} = -\infty$

$\lim_{x\to1^-} \frac{x^2}{x^2-1} = \frac{1}{0^-} = -\infty$ $\lim_{x\to1^+} \frac{x^2}{x^2-1} = \frac{1}{0^+} = \infty$

ABS: $x = -1$ $x = 1$

ASS: $y = kx + q$ $k = \frac{f(x)}{x}$ $f(x) = \frac{x^2}{x^2-1}$

$k: \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x}{x^2-1} = 0$

$q: \lim_{x\to\infty} (f(x) - kx) = \lim_{x\to\infty} \left(\frac{x^2}{x^2-1} - 0\cdot x\right) = \lim_{x\to\infty} \frac{x^2}{x^2-1} = 1$

ASS: $y = 0\cdot x + 1$

$y = 1$

$k: \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{x}{x^2-1} = 0$

$q: \lim_{x\to-\infty} (f(x) - kx) = \lim_{x\to-\infty} \left(\frac{x^2}{x^2-1} - 0\cdot x\right) = \lim_{x\to-\infty} \frac{x^2}{x^2-1} = 1$ } $y = 1$

④ a) $D(f) = \mathbb{R} - \{\pm 1\}$ ✓ $\forall x \in D(x): x \in D(x) \wedge -x \in D(x)$

b) $f(-x) = \frac{(-x)^2}{(-x)^2-1} = \frac{x^2}{x^2-1} = f(x)$ → fcia je PÁRNA

⑤ $O_y: f(0) = \frac{0^2}{0^2-1} = \frac{0}{-1} = 0$ $[0,0]$

Priesečník s osami: $[0,0]$

$O_x: 0 = \frac{x^2}{x^2-1} \Rightarrow x^2 = 0 \Rightarrow x = 0$ $[x;0]$

⑥ $f'(x) = \left(\frac{x^2}{x^2-1}\right)' = \frac{2x(x^2-1) - x^2 \cdot 2x}{(x^2-1)^2} = \ominus \frac{2x}{(x^2-1)^2}$ $f'(x)$

$\frac{-2x}{(x^2-1)^2} = 0 \Leftrightarrow 2x = 0$

$x = 0$

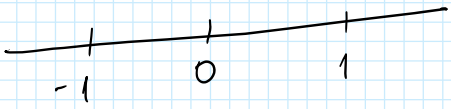
$f' = 0 : 0$

$\nexists f' : \pm 1$

$SB : 0$

	$(-\infty;-1)$	-1	$(-1,0)$	0	$(0,1)$	1	$(1,\infty)$
f'	+	X	+	SB	−	X	−
f	↗	X	↗	LOK MAX.	↘	X	↘

$[0;0]$



$f''(x) = \left(\frac{-2x}{(x^2-1)^2}\right)' = \frac{-2(x^2-1)^2 - (-2x)\,2(x^2-1)\cdot 2x}{(x^2-1)^4} = \frac{(x^2-1)[-2(x^2-1) + 8x^2]}{(x^2-1)^4}$

$f''(x) = \frac{2(3x^2+1)}{(x^2-1)^3}$

$f''(x) = 0$: nexistuje

$\nexists f''(x) : \pm 1$

IB → nexistuje

