RzM_7b

streda 11. novembra 2026 10:16

Pr $f(x) = \frac{x^3}{2(x+1)^2}$ ① $D(f) = \mathbb{R} - \{-1\}$

② $\lim_{x\to\infty} \frac{x^3}{2(x+1)^2} = \infty$ $\lim_{x\to-\infty} \frac{x^3}{2(x+1)^2} = -\infty$

③ ABS: $\lim_{x\to-1^-} \frac{x^3 \to -1}{2(x+1)^2 \to 0^+} = -\infty$ $\lim_{x\to-1^+} \frac{x^3 \to -1}{2(x+1)^2 \to 0^+} = -\infty$

$\xrightarrow{-1001\ -1\ -0.99}$

ABS: $x = -1$

ASS: $y = kx + q$ $f(x) = \frac{x^3}{2(x+1)^2}$

$k = \lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{x^2}{2(x+1)^2} = \frac{1}{2}$

$q = \lim_{x\to\pm\infty} (f(x) - kx) = \lim_{x\to\pm\infty} \left(\frac{x^3}{2(x+1)^2} - \frac{1}{2}x\right) = \lim_{x\to\pm\infty} \frac{x^3 - x^3 - 2x^2 - x}{2(x+1)^2} =$

$= \lim_{x\to\pm\infty} \frac{-2x^2 - x}{2(x+1)^2} = -1$

ASS: $y = \frac{1}{2}x - 1$

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④ ① $D(f) = \mathbb{R} - \{-1\} \to D(f)$ nie je symetrický vzhľadom na 0

fcia je ani párna ani nepárna

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⑤ O_y: $f(0) = \frac{0^3}{2(0+1)^2} = 0$ O_y: $[0,0]$

O_x: $0 = \frac{x^3}{2(x+1)^2} \Rightarrow x = 0$ O_x: $[0,0]$

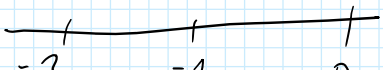
⑥ $f(x) = \frac{x^3}{2(x+1)^2}$

$$f'(x) = \frac{3x^2 \cdot 2(x+1)^2 - x^3 \cdot 4(x+1)}{4(x+1)^4} = \frac{(x+1)\left[6x^2(x+1) - 4x^3\right]}{4(x+1)^4} =$$

$$= \frac{6x^3 + 6x^2 - 4x^3}{4(x+1)^3} = \frac{2x^3 + 6x^2}{4(x+1)^3} = \frac{2x^2(x+3)}{4(x+1)^3} = \frac{x^2(x+3)}{2(x+1)^3}$$

$f'(x) = 0 : 0; -3$ → SB

$\nexists f'(x) : -1$



	$(-\infty;-3)$	-3	$(-3;-1)$	-1	$(-1;0)$	0	$(0;\infty)$
f'	+		−		+		+
f	↗	LMAX	↘		↗		↗
		$[-3; -\frac{27}{8}]$					

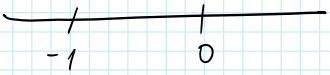
⑦ $$f''(x) = \left(\frac{x^3+3x^2}{2(x+1)^3}\right)' = \frac{(3x^2+6x)2(x+1)^3 - (x^3+3x^2)2\cdot 3(x+1)^2}{4(x+1)^6} =$$

$$= \frac{2(x+1)^2\left[(3x^2+6x)(x+1) - (x^3+3x^2)3\right]}{4(x+1)^6} = \frac{3x^3+6x^2+3x^2+6x-3x^3-9x^2}{2(x+1)^4} =$$

$$f''(x) = \frac{6x}{2(x+1)^4}$$

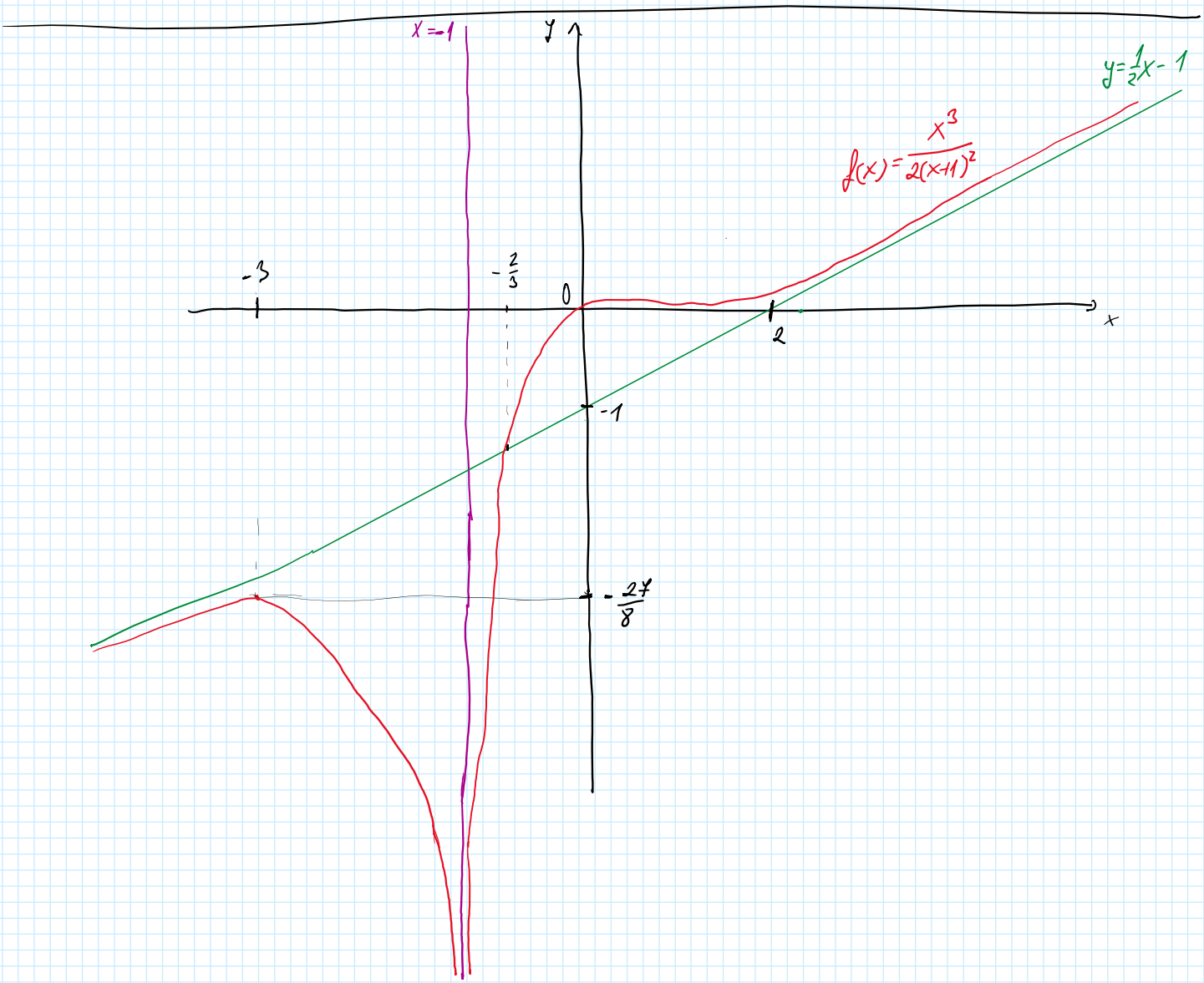
$f''(x) = 0 : 0$

$\nexists f''(x) : -1$



	$(-\infty;-1)$	-1	$(-1;0)$	0	$(0;\infty)$
$f''(x)$	−		−	IB	+
$f(x)$	∩		∩	[2;0]	∪

GRAF NA DALŠEJ STRANE ☺


x = -1
y
x
y = 1/2 x - 1
f(x) = x^3 / 2(x+1)^2
-3
-2/3
0
2
-1
-27/8