RZM_7a

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Pr: $f(x) = \frac{x}{x^2-1}$ ① $D(f) = \mathbb{R} - \{\pm 1\}$

② $\lim_{x\to\infty} \frac{x}{x^2-1} = 0$ $\lim_{x\to-\infty} \frac{x}{x^2-1} = 0$

③ ABS

$\lim_{x\to -1^-} \frac{x}{x^2-1} = -\infty$ ($x \to -1$, $x^2-1 \to 0^+$)

$\lim_{x\to -1^+} \frac{x}{x^2-1} = \infty$ ($x \to -1$, $x^2-1 \to 0^-$)

$\lim_{x\to 1^-} \frac{x}{x^2-1} = -\infty$ ($x \to 1$, $x^2-1 \to 0^-$)

$\lim_{x\to 1^+} \frac{x}{x^2-1} = \infty$ ($x \to 1$, $x^2-1 \to 0^+$)

ABS: $x = -1$ $x = 1$

ASS: $y = kx + q$ $f(x) = \frac{x}{x^2-1}$

$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{1}{x^2-1} = 0$

$q = \lim_{x\to\infty} (f(x) - kx) = \lim_{x\to\infty} \left(\frac{x}{x^2-1} - 0\cdot x\right) = \lim_{x\to\infty} \frac{x}{x^2-1} = 0$

$y = 0$

$k = \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{1}{x^2-1} = 0$

$q = \lim_{x\to-\infty} (f(x) - kx) = \lim_{x\to\infty} \left(\frac{x}{x^2-1} - 0\cdot x\right) = \lim_{x\to-\infty} \frac{x}{x^2-1} = 0$

$y = 0$

④ ① $D(f) = \mathbb{R} - \{\pm 1\}$ ✓ → $D(f)$ je symetrický vzhľadom na 0

② $f(-x) = \frac{-x}{(-x)^2-1} = -\frac{x}{x^2-1} = -f(x)$ → fcia je nepárna

⑤ $O_y: y = f(0) = \frac{0}{0^2-1} = 0$ $O_y: [0;0]$

$O_x: 0 = \frac{x}{x^2-1}$ → $x = 0$ $O_x: [0,0]$

⑥ $f(x) = \frac{x}{x^2-1}$

$f'(x) = \frac{x^2-1 - x\cdot 2x}{(x^2-1)^2} = \ominus \frac{(x^2+1)^+}{(x^2-1)^2+}$

$f'(x) < 0$ pre $\forall x \in D(f) \Rightarrow$

$\Rightarrow$ na celom $D(f)$ je $\searrow$

$f'(x) = 0$: nemá $\Rightarrow$ nemá SB $\Rightarrow$ nemá extrém

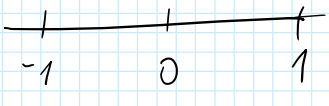
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⑦ $f''(x) = \left(\frac{-x^2-1}{(x^2-1)^2}\right)' = \frac{-2x(x^2-1)^2 - (-x^2-1)\cdot 2(x^2-1)\cdot 2x}{(x^2-1)^4} = \frac{(x^2-1)[-2x(x^2-1)+(x^2+1)4x]}{(x^2-1)^{+3}} =$

$= \frac{-2x^3+2x+4x^3+4x}{(x^2-1)^3} = \frac{2x(x^2+3)^+}{(x^2-1)^3}$

$f''(x) = 0$: 0

$\nexists f''(x)$: ± 1



	$(-\infty;-1)$	-1	$(-1;0)$	0	$(0;1)$	1	$(1;\infty)$
f''	$\overset{=}{+}$ $-$	∄	$\overset{=}{-}$ $+$	IB	$\overset{=}{+}$ $-$	∄	$\overset{=}{+}$ $+$
f	∩	∄	∪	[0;0]	∩	∄	∪

