

Technical University of Košice
Faculty of Electrical Engineering and Informatics



Applied Statistics

Formula Sheet

100(1 - α)% Confidence Intervals

μ (known σ^2), $X \sim N(\mu, \sigma^2)$

two-sided	lower one-sided	upper one-sided
$\left\langle \bar{x} - u_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}; \bar{x} + u_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \right\rangle$	$\left\langle \bar{x} - u_{1-\alpha} \frac{\sigma}{\sqrt{n}}; \infty \right\rangle$	$\left(-\infty; \bar{x} + u_{1-\alpha} \frac{\sigma}{\sqrt{n}} \right)$

μ (unknown σ^2), $X \sim N(\mu, \sigma^2)$

two-sided	lower one-sided	upper one-sided
$\left\langle \bar{x} - t_{1-\frac{\alpha}{2}}(n-1) \frac{s}{\sqrt{n}}; \bar{x} + t_{1-\frac{\alpha}{2}}(n-1) \frac{s}{\sqrt{n}} \right\rangle$	$\left\langle \bar{x} - t_{1-\alpha}(n-1) \frac{s}{\sqrt{n}}; \infty \right\rangle$	$\left(-\infty; \bar{x} + t_{1-\alpha}(n-1) \frac{s}{\sqrt{n}} \right)$

σ^2 (known μ), $X \sim N(\mu, \sigma^2)$

two-sided	lower one-sided	upper one-sided
$\left\langle \frac{n s_0^2}{\chi_{1-\frac{\alpha}{2}}^2(n)}; \frac{n s_0^2}{\chi_{\frac{\alpha}{2}}^2(n)} \right\rangle$	$\left\langle \frac{n s_0^2}{\chi_{1-\alpha}^2(n)}; \infty \right\rangle$	$\left(0; \frac{n s_0^2}{\chi_{\alpha}^2(n)} \right)$

σ^2 (unknown μ), $X \sim N(\mu, \sigma^2)$

two-sided	lower one-sided	upper one-sided
$\left\langle \frac{(n-1) s^2}{\chi_{1-\frac{\alpha}{2}}^2(n-1)}; \frac{(n-1) s^2}{\chi_{\frac{\alpha}{2}}^2(n-1)} \right\rangle$	$\left\langle \frac{(n-1) s^2}{\chi_{1-\alpha}^2(n-1)}; \infty \right\rangle$	$\left(0; \frac{(n-1) s^2}{\chi_{\alpha}^2(n-1)} \right)$

λ , $X \sim \text{Ex}(\lambda)$

two-sided	lower one-sided	upper one-sided
$\left\langle \frac{2n\bar{x}}{\chi_{1-\frac{\alpha}{2}}^2(2n)}; \frac{2n\bar{x}}{\chi_{\frac{\alpha}{2}}^2(2n)} \right\rangle$	$\left\langle \frac{2n\bar{x}}{\chi_{1-\alpha}^2(2n)}; \infty \right\rangle$	$\left(0; \frac{2n\bar{x}}{\chi_{\alpha}^2(2n)} \right)$

One-Sample Parametric Tests

Test μ (known σ^2), $V_n \sim N(\mu, \sigma^2)$

H_0	H_1	Test statistic	K_α
$\mu = \mu_0$	$\mu \neq \mu_0$	$Z = \frac{\bar{x} - \mu_0}{\sigma} \sqrt{n}$	$(-\infty; -u_{1-\frac{\alpha}{2}}) \cup (u_{1-\frac{\alpha}{2}}; \infty)$
$\mu = \mu_0$	$\mu > \mu_0$		$(u_{1-\alpha}; \infty)$
$\mu = \mu_0$	$\mu < \mu_0$		$(-\infty; -u_{1-\alpha})$

Test μ (unknown σ^2), $V_n \sim N(\mu, \sigma^2)$

H_0	H_1	Test statistic	K_α
$\mu = \mu_0$	$\mu \neq \mu_0$	$t = \frac{\bar{x} - \mu_0}{s} \sqrt{n}$	$(-\infty; -t_{1-\frac{\alpha}{2}}(n-1)) \cup (t_{1-\frac{\alpha}{2}}(n-1); \infty)$
$\mu = \mu_0$	$\mu > \mu_0$		$(t_{1-\alpha}(n-1); \infty)$
$\mu = \mu_0$	$\mu < \mu_0$		$(-\infty; -t_{1-\alpha}(n-1))$

Test σ^2 (known μ), $V_n \sim N(\mu, \sigma^2)$

H_0	H_1	Test statistic	K_α
$\sigma^2 = \sigma_0^2$	$\sigma^2 \neq \sigma_0^2$	$\chi^2 = \frac{n s_0^2}{\sigma_0^2}$	$(0; \chi_{\frac{\alpha}{2}}^2(n)) \cup (\chi_{1-\frac{\alpha}{2}}^2(n); \infty)$
$\sigma^2 = \sigma_0^2$	$\sigma^2 > \sigma_0^2$		$(\chi_{1-\alpha}^2(n); \infty)$
$\sigma^2 = \sigma_0^2$	$\sigma^2 < \sigma_0^2$		$(0; \chi_\alpha^2(n))$

Test σ^2 (unknown μ), $V_n \sim N(\mu, \sigma^2)$

H_0	H_1	Test statistic	K_α
$\sigma^2 = \sigma_0^2$	$\sigma^2 \neq \sigma_0^2$	$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$	$(0; \chi_{\frac{\alpha}{2}}^2(n-1)) \cup (\chi_{1-\frac{\alpha}{2}}^2(n-1); \infty)$
$\sigma^2 = \sigma_0^2$	$\sigma^2 > \sigma_0^2$		$(\chi_{1-\alpha}^2(n-1); \infty)$
$\sigma^2 = \sigma_0^2$	$\sigma^2 < \sigma_0^2$		$(0; \chi_\alpha^2(n-1))$

Test λ , $V_n \sim \text{Ex}(\lambda)$

H_0	H_1	Test statistic	K_α
$\lambda = \lambda_0$	$\lambda \neq \lambda_0$	$\chi^2 = \frac{2n\bar{x}}{\lambda_0}$	$(0; \chi_{\frac{\alpha}{2}}^2(2n)) \cup (\chi_{1-\frac{\alpha}{2}}^2(2n); \infty)$
$\lambda = \lambda_0$	$\lambda > \lambda_0$		$(\chi_{1-\alpha}^2(2n); \infty)$
$\lambda = \lambda_0$	$\lambda < \lambda_0$		$(0; \chi_\alpha^2(2n))$

Two-Sample Parametric Tests, $V_n \sim N(\mu, \sigma^2)$

Fisher's F-test

H_0	H_1	Test statistic	K_α
$\sigma_1^2 = \sigma_2^2$	$\sigma_1^2 \neq \sigma_2^2$	$F = \frac{s_1^2}{s_2^2}$	$(0; F_{\frac{\alpha}{2}}(n_1 - 1, n_2 - 1)) \cup (F_{1-\frac{\alpha}{2}}(n_1 - 1, n_2 - 1); \infty)$
$\sigma_1^2 = \sigma_2^2$	$\sigma_1^2 > \sigma_2^2$		$(F_{1-\alpha}(n_1 - 1, n_2 - 1); \infty)$
$\sigma_1^2 = \sigma_2^2$	$\sigma_1^2 < \sigma_2^2$		$(0; F_\alpha(n_1 - 1, n_2 - 1))$

Two-sample Z-test (variances known σ_1^2, σ_2^2)

H_0	H_1	Test statistic	K_α
$\mu_1 = \mu_2$	$\mu_1 \neq \mu_2$	$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$	$(-\infty; -u_{1-\frac{\alpha}{2}}) \cup (u_{1-\frac{\alpha}{2}}; \infty)$
$\mu_1 = \mu_2$	$\mu_1 > \mu_2$		$(u_{1-\alpha}; \infty)$
$\mu_1 = \mu_2$	$\mu_1 < \mu_2$		$(-\infty; -u_{1-\alpha})$

Two-sample t-test (variances unknown σ_1^2, σ_2^2 and assuming $\sigma_1^2 = \sigma_2^2$)

H_0	H_1	Test statistic	K_α
$\mu_1 = \mu_2$	$\mu_1 \neq \mu_2$	$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}} \sqrt{\frac{n_1 n_2 k}{n_1 + n_2}}$	$(-\infty; -t_{1-\frac{\alpha}{2}}(k)) \cup (t_{1-\frac{\alpha}{2}}(k); \infty)$
$\mu_1 = \mu_2$	$\mu_1 > \mu_2$		$(t_{1-\alpha}(k); \infty)$
$\mu_1 = \mu_2$	$\mu_1 < \mu_2$		$(-\infty; -t_{1-\alpha}(k))$

$k = (n_1 + n_2 - 2)$

Aspin-Welch test (variances unknown σ_1^2, σ_2^2 and assuming $\sigma_1^2 \neq \sigma_2^2$)

H_0	H_1	Test statistic	K_α
$\mu_1 = \mu_2$	$\mu_1 \neq \mu_2$	$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$	$(-\infty; -t_{1-\frac{\alpha}{2}}(m)) \cup (t_{1-\frac{\alpha}{2}}(m); \infty)$
$\mu_1 = \mu_2$	$\mu_1 > \mu_2$		$(t_{1-\alpha}(m); \infty)$
$\mu_1 = \mu_2$	$\mu_1 < \mu_2$		$(-\infty; -t_{1-\alpha}(m))$

$m = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$

Paired t-test

H_0	H_1	Test statistic	K_α
$\mu_1 = \mu_2$	$\mu_1 \neq \mu_2$	$t = \frac{\bar{d}}{s_d} \sqrt{n}$, where $\bar{d} = \frac{1}{n} \sum_{i=1}^n d_i = \frac{1}{n} \sum_{i=1}^n (x_{1i} - x_{2i})$, $s_d^2 = \frac{1}{n-1} \sum_{i=1}^n (d_i - \bar{d})^2$	$(-\infty; -t_{1-\frac{\alpha}{2}}(n-1)) \cup (t_{1-\frac{\alpha}{2}}(n-1); \infty)$
$\mu_1 = \mu_2$	$\mu_1 > \mu_2$		$(t_{1-\alpha}(n-1); \infty)$
$\mu_1 = \mu_2$	$\mu_1 < \mu_2$		$(-\infty; -t_{1-\alpha}(n-1))$

Symmetry Tests

Sign Test

For a small sample V_n :

H_0	H_1	Test statistic	K_α
$\tilde{x} = x_0$	$\tilde{x} \neq x_0$	$T = \min\{Z^+, Z^-\}$	$(-\infty; k_1(\alpha, n_0)) \cup (k_2(\alpha, n_0); \infty)$

Z^+ number of values $d_i = x_i - x_0$ with a positive sign

Z^- number of values $d_i = x_i - x_0$ with a negative sign

n_0 number of non-zero values $d_i = x_i - x_0$

$k_1(\alpha, n_0)$, $k_2(\alpha, n_0)$ are the critical values of the test

For a large sample V_n :

H_0	H_1	Test statistic	K_α
$\tilde{x} = x_0$	$\tilde{x} \neq x_0$	$Z = \frac{2T - n_0}{\sqrt{n_0}}$	$(-\infty; -u_{1-\frac{\alpha}{2}}) \cup (u_{1-\frac{\alpha}{2}}; \infty)$
$\tilde{x} = x_0$	$\tilde{x} > x_0$		$(u_{1-\alpha}; \infty)$
$\tilde{x} = x_0$	$\tilde{x} < x_0$		$(-\infty; -u_{1-\alpha})$

Wilcoxon Two-Sample Test

For a small sample V_n :

H_0	H_1	Test statistic	K_α
$F(x) = G(x)$	$F(x) \neq G(x)$	$U = \min\{U_1, U_2\}$, where $U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - T_1$, $U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - T_2$	$(-\infty; U_\alpha)$

T_1 is the sum of ranks in sample V_{n_1}

T_2 is the sum of ranks in sample V_{n_2}

U_α is the critical value of the test

For a large sample V_n :

H_0	H_1	Test statistic	K_α
$F(x) = G(x)$	$F(x) \neq G(x)$	$U_0 = \frac{U_1 - \frac{n_1 n_2}{2}}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}}$	$(-\infty; -u_{1-\frac{\alpha}{2}}) \cup (u_{1-\frac{\alpha}{2}}; \infty)$

Goodness-of-Fit Tests

Pearson Goodness-of-Fit Test

H_0	H_1	Test statistic	K_α
$F(x) = G(x)$	$F(x) \neq G(x)$	$\chi^2 = \sum_{j=1}^k \frac{(n_j - np_j)^2}{np_j}$	$(\chi^2_{1-\alpha}(k - r - 1); \infty)$

r is the number of parameters estimated when determining G

k is the number of classes

Kolmogorov–Smirnov Test

H_0	H_1	Test statistic	K_α
$F(x) = G(x)$	$F(x) \neq G(x)$	$D = \max\{\max(d_j^-), \max(d_j^+)\}$, where $d_i^- = F_n(x_{j-1}) - G(x_j) $, $d_i^+ = F_n(x_j) - G(x_j) $	$(D_\alpha(n); \infty)$

$D_\alpha(n)$ is the critical value; for large samples V_n ($n > 100$) it is approximated by $D_\alpha(n) \doteq \sqrt{\frac{1}{2n} \ln \frac{2}{\alpha}}$

Anderson–Darling Test

H_0	H_1	Test statistic	K_α
$F(x) = G(x)$	$F(x) \neq G(x)$	$AD = \frac{1}{n} \sum_{i=1}^n (1 - 2i) [\ln G(x_{(i)}) + \ln (1 - G_{(n+1-i)})] - n$	$(AD_\alpha; \infty)$

AD_α is the critical value of the test

if $G(x) \sim N(\mu, \sigma^2)$, small sample $AD_\alpha \approx \frac{k(\alpha)}{1 + \frac{0,75}{n} + \frac{2,25}{n^2}}$, where

α	0,005	0,01	0,0025	0,05	0,1
$k(\alpha)$	1,159	1,035	0,873	0,752	0,631

if $G(x) \sim Ex(\lambda)$

α	0,005	0,01	0,0025	0,05	0,1
AD_α	2,244	1,959	1,591	1,321	1,162

Shapiro–Wilk Test of Normality

H_0	H_1	Test statistic	K_α
$F(x) = G(x)$	$F(x) \neq G(x)$	$W = \frac{\left(\sum_{i=1}^m a_{i,n} (x_{(n-i+1)} - x_{(i)}) \right)^2}{\sum_{i=1}^n (x_{(i)} - \bar{x})^2}$	$(-\infty; W_\alpha(n))$

$W_\alpha(n)$ is the critical value of the test

Outlier Tests

Grubbs' Test

H_0	H_1	Test statistic	K_α
value $x_{(1)}$ is not an outlier	value $x_{(1)}$ is an outlier	$T(1) = \frac{\bar{x} - x_{(1)}}{s} \sqrt{\frac{n}{n-1}}$	$(T_\alpha(n); \infty)$
value $x_{(n)}$ is not an outlier	value $x_{(n)}$ is an outlier	$T(n) = \frac{x_{(n)} - \bar{x}}{s} \sqrt{\frac{n}{n-1}}$	

$T_\alpha(n)$ is the critical value of the test

Dixon's Test

H_0	H_1	Test statistic	K_α
value $x_{(1)}$ is not an outlier	value $x_{(1)}$ is an outlier	$Q(1) = \frac{x_{(2)} - x_{(1)}}{x_{(n)} - x_{(1)}}$	$(Q_\alpha(n); \infty)$
value $x_{(n)}$ is not an outlier	value $x_{(n)}$ is an outlier	$Q(n) = \frac{x_{(n)} - x_{(n-1)}}{x_{(n)} - x_{(1)}}$	

$Q_\alpha(n)$ is the critical value of the test

Tests of Zero Correlation

Test of Zero Correlation

H_0	H_1	Test statistic	K_α
$\rho = 0$	$\rho \neq 0$	$t = \frac{r_{xy} \sqrt{n-2}}{\sqrt{1-r_{xy}^2}}$	$(-\infty; -t_{1-\frac{\alpha}{2}}(n-2)) \cup (t_{1-\frac{\alpha}{2}}(n-2); \infty)$
$\rho = 0$	$\rho > 0$		$(t_{1-\alpha}(n-2); \infty)$
$\rho = 0$	$\rho < 0$		$(-\infty; -t_{1-\alpha}(n-2))$

r_{xy} is the sample Pearson correlation coefficient

$T_\alpha(n)$ is the critical value of the test

Spearman Independence Test

For a small sample V_n :

H_0	H_1	Test statistic	K_α
$r_S = 0$	$r_S \neq 0$	r_S	$(-\infty; -r_\alpha(n)) \cup (r_\alpha(n); \infty)$

r_S is Spearman's rank correlation coefficient

$r_\alpha(n)$ is the critical value of the test

For a large sample V_n :

H_0	H_1	Test statistic	K_α
$r_S = 0$	$r_S \neq 0$	$t = \frac{r_S \sqrt{n-2}}{\sqrt{1-r_S^2}}$	$(-\infty; -t_{1-\frac{\alpha}{2}}(n-2)) \cup (t_{1-\frac{\alpha}{2}}(n-2); \infty)$

Tests in Regression Analysis

Regression Model Significance Test

H_0	H_1	Test statistic	K_α
regression model is not statistically significant	regression model is statistically significant	$F = \frac{(n-m) \sum_{i=1}^n (\hat{y}_i - \bar{y})^2}{(m-1) \sum_{i=1}^n (y_i - \hat{y}_i)^2}$	$(F_{1-\alpha}(m-1, n-m); \infty)$

m is the number of estimated regression-function parameters

Test of a Regression Parameter Against a Known Constant

H_0	H_1	Test statistic	K_α
$\beta_i = b$ $\beta_i = b$ $\beta_i = b$	$\beta_i \neq b$ $\beta_i > b$ $\beta_i < b$	$t = \frac{b_i - b}{s(b_i)}$	$(-\infty; -t_{1-\frac{\alpha}{2}}(n-2)) \cup (t_{1-\frac{\alpha}{2}}(n-2); \infty)$ $(t_{1-\alpha}(n-2); \infty)$ $(-\infty; -t_{1-\alpha}(n-2))$

$s(b_i)$ is the sample standard deviation of the estimated parameter b_i

Goldfeld-Quandt Test

H_0	H_1	Test statistic	K_α
$\sigma_d^2 = \sigma_h^2$	$\sigma_d^2 > \sigma_h^2$	$F = \frac{SSE_d}{SSE_h} \frac{n_h - m}{n_d - m} = \frac{MSE_d}{MSE_h}$	$(F_{1-\alpha}(n_d - m, n_h - m); \infty)$

Test of Error Randomness

H_0	H_1	Test statistic	K_α
errors are random	errors are not random	$U = \frac{Z - \frac{2n-4}{3}}{\sqrt{\frac{16n-29}{90}}}$	$(-\infty; -u_{1-\frac{\alpha}{2}}) \cup (u_{1-\frac{\alpha}{2}}; \infty)$

Z is the total number of turning points in the sequence $e_1, e_2, \dots, e_n$