

PRÍKLAD: VÝPOČÍTANIE

$$a) \quad \left(-\frac{1}{2} + 2i\right) \cdot 2i - 3 \cdot (1+i) = -i + 4i^2 - 3 - 3i = \underline{\underline{-7 - 4i}}$$

$$b) \quad \frac{2-3i}{3-2i} + \frac{1}{13} (1+5i) = \frac{2-3i}{3-2i} \cdot \frac{3+2i}{3+2i} + \frac{1}{13} (1+5i) = \frac{6+4i-9i+6}{9+4} + \frac{1}{13} + \frac{5}{13}i =$$

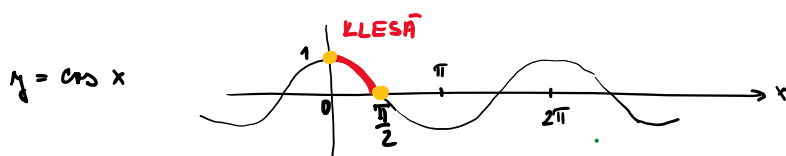
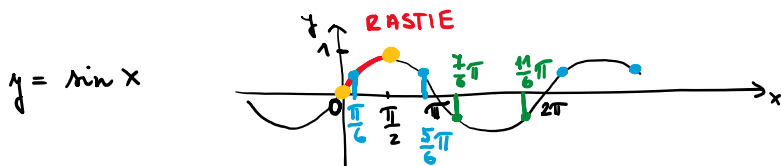
$$= \frac{12-5i}{13} + \frac{1}{13} + \frac{5}{13}i = \frac{12}{13} - \frac{5}{13}i + \frac{1}{13} + \frac{5}{13}i = \underline{\underline{1}}$$

$$c) \quad (-2+3i)^2 + (1+3i)(1-3i) = 4 - 12i + 9i^2 + 1 - 9i^2 = \underline{\underline{5 - 12i}}$$

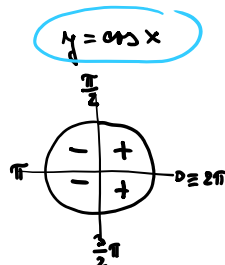
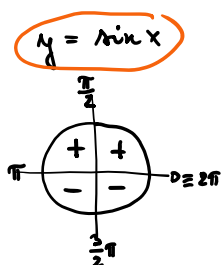
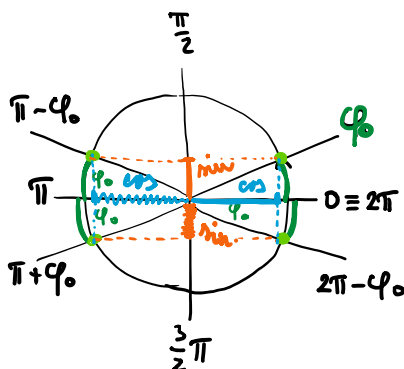
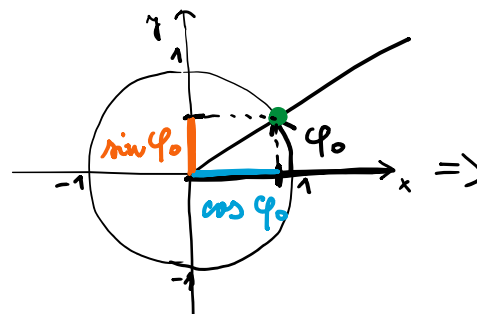
$$d) \quad \frac{5+2i}{-2+i} + \frac{2}{5}i(3-2i) - \frac{2}{5}i^5 = \frac{5+2i}{-2+i} \cdot \frac{-2-i}{-2-i} + \frac{6}{5}i + \frac{4}{5} - \frac{2}{5}i^2 \cdot i =$$

$$= \frac{-10-5i-4i+2}{4+1} + \frac{6}{5}i + \frac{4}{5} - \frac{2}{5}i = \frac{-8-9i}{5} + \frac{4}{5}i + \frac{4}{5} = \underline{\underline{-\frac{4}{5} - i}}$$

KRÁTKY NÁVRAT K STREDNEJ ŠKOLE ☺



JEDNOTKOVÁ KRUŽNICA



	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

$$\varphi = \frac{131}{3}\pi \Rightarrow \text{UPRAVÍME POMOCOU MENŠIEHO UHLA}$$

$$\frac{131}{3}\pi : 2\pi \Rightarrow \left(\frac{\frac{131}{3}\pi}{\frac{2\pi}{1}} \right) = \frac{131\cancel{\pi}}{6\cancel{\pi}} = \left[\frac{131}{6} \right]$$

KOĽKO KRÁT PREBEHNEM CELÚ KRUŽNICU

$$131 : 6 = (21) \Rightarrow \text{KRÁT PREBEHNEM CELÚ KRUŽNICU}$$

IV. kvadr.

ZVÝŠOK $\overset{11}{\boxed{5}} \rightarrow$ DOSTANEM SA DO $\frac{5}{3}\pi \Rightarrow \varphi = \frac{131}{3}\pi \Rightarrow \varphi = \frac{5}{3}\pi$

PRÍKLAD: URČME HODNOTY F-CH

a) $\cos \frac{131}{3}\pi = \cos \left(\frac{5}{3}\pi \right) = + \frac{1}{2}$

$$131 : 6 = 21$$

zv. 5

IV. KVADR.

b) $\sin \frac{97}{6}\pi = \sin \left(\frac{1}{6}\pi \right) = + \frac{1}{2}$

$$97 : 12 = 8$$

zv. 1

I. KVADR.

c) $\cos \frac{35}{4}\pi = \cos \left(\frac{3}{4}\pi \right) = - \frac{\sqrt{2}}{2}$

$$35 : 8 = 4$$

zv. 3

II. KVADR.

d) $\sin \frac{28}{6}\pi = \sin \frac{14}{3}\pi = \sin \left(\frac{2}{3}\pi \right) = + \frac{\sqrt{3}}{2}$

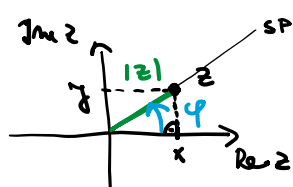
$$14 : 6 = 2$$

zv. 2

II. KVADR.

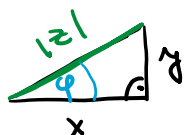
PREPIS KČ Z ALGEBRAICKÉHO TVARU DO GONIOMETRICKÉHO A EXPONENCIÁLNÉHO TVARU

KČ MÔŽEME ZNÁZORNIŤ V GAUSSEVEJ ROVINE POMOČOU REÁLNEJ A IMAGINÁRNEJ ZLOŽKY (ALG. TVAR), ALE EXISTUJE ZNÁZORNENIE POMOČOU MODULU A AMPLITÚDY (GEOM.; EXPON. TVAR)



$$|z| - \text{MODUL} \quad ; \quad |z| = \sqrt{x^2 + y^2}$$

φ - AMPLITÚDA - UHOL MEDZI SPRIEVODIČOM KČ A Kladnou Časťou REÁLNEJ OS



$$\begin{aligned} \cos \varphi &= \frac{x}{|z|} \Rightarrow x = |z| \cdot \cos \varphi \\ \sin \varphi &= \frac{y}{|z|} \Rightarrow y = |z| \cdot \sin \varphi \end{aligned} \quad \} \Rightarrow$$

$$\text{KČ } z = x + iy = \underline{|z| \cdot \cos \varphi} + i \underline{|z| \cdot \sin \varphi} = \underline{|z| \cdot (\cos \varphi + i \sin \varphi)}$$

↓
GONIOMETRICKÝ TVAR KČ

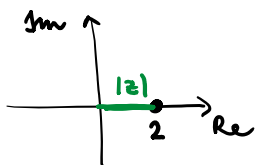
$$\text{PLATÍ: } \boxed{\cos \varphi + i \sin \varphi = e^{i \cdot \varphi}} \rightarrow \text{EULEROV VZŤAH} \Rightarrow$$

$$z = |z| \cdot (\cos \varphi + i \sin \varphi) = \underline{|z| \cdot e^{i \varphi}}$$

↓
EXPONENCIÁLNY TVAR KČ

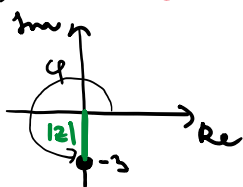
PRÍKLAD: PREPÍŠME DO GONIOMET. A EXPONENC. TVARU KČ

a) $z_1 = \overset{x}{2} + \overset{y}{0}i$



$$\begin{aligned} |z| &= 2 \\ \varphi &= 0 \end{aligned} \quad \} \quad \begin{aligned} z_1 &= 2 (\cos 0 + i \sin 0) \\ z_1 &= 2 \cdot e^{i \cdot 0} \end{aligned}$$

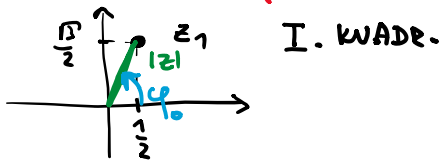
b) $z_2 = \overset{x}{0} + \overset{y}{-3}i$



$$\begin{aligned} |z| &= 3 \\ \varphi &= \frac{3}{2}\pi \end{aligned} \quad \} \quad \begin{aligned} z_2 &= 3 (\cos \frac{3}{2}\pi + i \sin \frac{3}{2}\pi) \\ z_2 &= 3 \cdot e^{i \cdot \frac{3}{2}\pi} \end{aligned}$$

PRÍKLAD: PREPÍŠTE DO KONDM. A EXPON. TVARU KE-

a) $z_1 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$



$$|z| = \sqrt{x^2 + y^2}$$

$$|z| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \underline{\underline{1}}$$

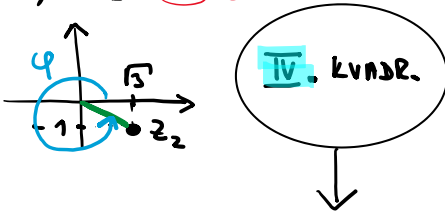
$$\cos \varphi_0 = \frac{|x|}{|z|} \leftarrow \text{ABSOLUTNÁ MODNOSTA}$$

$$\cos \varphi_0 = \frac{\frac{1}{2}}{1} = \frac{1}{2} \Rightarrow \varphi_0 = \frac{\pi}{3}$$

$$z_1 = 1 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$z_1 = 1 \cdot e^{i \cdot \frac{\pi}{3}}$$

b) $z_2 = \sqrt{3} - i$



$$\varphi = 2\pi - \varphi_0$$

$$\varphi = 2\pi - \frac{\pi}{6}$$

$$\varphi = \frac{11}{6}\pi$$

$$|z| = \sqrt{x^2 + y^2}$$

$$|z| = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{3 + 1} = \underline{\underline{2}}$$

$$\cos \varphi_0 = \frac{|x|}{|z|}$$

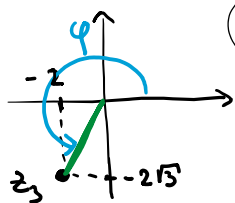
I. kv.

$$\cos \varphi_0 = \frac{\sqrt{3}}{2} \Rightarrow \varphi_0 = \frac{\pi}{6}$$

$$z_2 = 2 \left(\cos \frac{11}{6}\pi + i \sin \frac{11}{6}\pi \right)$$

$$z_2 = 2 \cdot e^{i \cdot \frac{11}{6}\pi}$$

$$c) \quad z_3 = -2 - 2\sqrt{3}i$$



π . kvadr.

$$\varphi = \pi + \varphi_0$$

$$\varphi = \pi + \frac{\pi}{3}$$

$$\varphi = \frac{4}{3}\pi$$

$$|z| = \sqrt{x^2 + y^2}$$

$$|z| = \sqrt{(-2)^2 + (-2\sqrt{3})^2} = \sqrt{4 + 12} = \underline{4}$$

$$\cos \varphi_0 = \frac{|x|}{|z|}$$

$$\cos \varphi_0 = \frac{2}{4} = \frac{1}{2} \Rightarrow \varphi_0 = \frac{\pi}{3}$$

$$z_3 = 4 \left(\cos \frac{4}{3}\pi + i \sin \frac{4}{3}\pi \right)$$

$$z_3 = 4 \cdot e^{i \cdot \frac{4}{3}\pi}$$

OPERÁCIE S KČ V GONIOMETRICKOM TVARE

SÚČIN KČ

- ROBÍME TAK, ĽE MODULY VYNÁSOŽÍME A AMPLITÚDY SČÍTAME

$$\text{NECH } z_1 = |z_1| (\cos \varphi_1 + i \sin \varphi_1)$$

$$z_2 = |z_2| (\cos \varphi_2 + i \sin \varphi_2)$$

$$\text{SÚČIN } z_1 \cdot z_2 = |z_1| \cdot |z_2| \cdot \left[\cos(\varphi_1 + \varphi_2) + i \sin(\varphi_1 + \varphi_2) \right]$$

$$\text{PRÍKLAD: VYPOČÍTAJTE SÚČIN } z_1 \cdot z_2 ; \quad z_1 = 2 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$$

$$z_2 = \frac{3}{2} \cdot (\cos \pi + i \sin \pi)$$

$$z_1 \cdot z_2 = 2 \cdot \frac{3}{2} \left[\cos \left(\frac{\pi}{2} + \pi \right) + i \sin \left(\frac{\pi}{2} + \pi \right) \right] = 3 \left(\cos \frac{3}{2}\pi + i \sin \frac{3}{2}\pi \right)$$

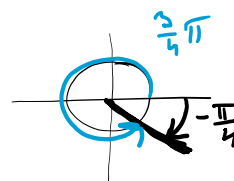
PODIEL KČ

- ROBÍME TAK, ĽE MODULY PREDELIAME A AMPLITÚDY ODČÍTAME

$$\text{NECH } z_1 = |z_1| (\cos \varphi_1 + i \sin \varphi_1)$$

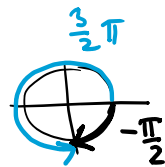
$$z_2 = |z_2| (\cos \varphi_2 + i \sin \varphi_2)$$

$$\text{PODIEL } \frac{z_1}{z_2} = \frac{|z_1|}{|z_2|} \left[\cos(\varphi_1 - \varphi_2) + i \sin(\varphi_1 - \varphi_2) \right]$$



PRÍKLAD: VYPOČÍTANIE PODIEL $\frac{z_2}{z_1}$ a $\frac{z_1}{z_2}$; $z_1 = 2(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})$
 $z_2 = \frac{3}{2} \cdot (\cos \pi + i \sin \pi)$

$$\frac{z_2}{z_1} = \frac{\frac{3}{2}}{\frac{2}{1}} \left[\cos \left(\pi - \frac{\pi}{2} \right) + i \sin \left(\pi - \frac{\pi}{2} \right) \right] = \frac{3}{4} \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$$



$$\frac{z_1}{z_2} = \frac{2}{\frac{3}{2}} \left[\cos \left(\frac{\pi}{2} - \pi \right) + i \sin \left(\frac{\pi}{2} - \pi \right) \right] = \frac{4}{3} \left[\cos \left(-\frac{\pi}{2} \right) + i \sin \left(-\frac{\pi}{2} \right) \right] = \frac{4}{3} \left[\cos \frac{3}{2}\pi + i \sin \frac{3}{2}\pi \right]$$