# INTEGROVANIE ROZKLADOM A ÚPRAVOU

- INTEGRAČNÚ F-CIU ROZLOŽÍME NA SÚČET, ROZDIEL JEDNODUCHÝCH F-CIÍ

VETA: NECH K F-CIÁM $f$ A $g$ EXISTUJÚ PRIMITÍVNE F-CIE A $k_1, k_2 \in \mathbb{R}$.
POTOM EXISTUJE PRIMITÍVNA F-CIA K F-CII $k_1 \cdot f + k_2 \cdot g$ A PLATÍ

$$\int \left(k_1 \cdot f(x) + k_2 \cdot g(x)\right) dx = k_1 \cdot \int f(x)\, dx + k_2 \cdot \int g(x)\, dx$$

## ZÁKLADNÉ VZORCE INTEGROVANIA

$\int a\, dx = ax + c \quad ; a \in \mathbb{R}$

$\int x^a\, dx = \frac{x^{a+1}}{a+1} + c$

$\int a^x\, dx = \frac{a^x}{\ln a} + c$

$\int e^x\, dx = e^x + c$

$\int \frac{1}{x}\, dx = \ln|x| + c$

$\int \cos x\, dx = \sin x + c$

$\int \sin x\, dx = -\cos x + c$

$\int \frac{1}{\cos^2 x}\, dx = \operatorname{tg} x + c$

$\int \frac{1}{\sin^2 x}\, dx = -\operatorname{cotg} x + c$

$\int \frac{f'(x)}{f(x)}\, dx = \ln|f(x)| + c$

PRÍKLAD: VYPOČÍTAJME INTEGRÁL

1) $\int (9x^2 - 2x + 1)\, dx = \int 9x^2 dx - \int 2x\, dx + \int 1\, dx = 9\int x^2 dx - 2\int x\, dx + \int 1\, dx =$

$= \cancel{9} \cdot \frac{x^3}{\cancel{3}} - \cancel{2} \cdot \frac{x^2}{\cancel{2}} + x + C = 3x^3 - x^2 + x + c$

2) $\int \left(x^{-6} - 3x^{\frac{1}{2}} + 5x^{-\frac{2}{3}}\right) dx = \frac{x^{-5}}{-5} - \cancel{3}\frac{x^{\frac{3}{2}}}{\frac{\cancel{3}}{2}} + 5 \cdot \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + c = \frac{-1}{5x^5} - 2\sqrt{x^3} + 15\sqrt[3]{x} + c$

3) $\int \left(\frac{1}{x^6} - 3\sqrt{x} + \frac{5}{\sqrt[3]{x^2}}\right) dx = \int \left(x^{-6} - 3x^{\frac{1}{2}} + 5x^{-\frac{2}{3}}\right) dx =$

$x^{-6} \quad -3x^{\frac{1}{2}} \quad +5 \cdot x^{-\frac{2}{3}}$

4) $\int 3x^2 \cdot (x^5 - 2x + 1)\, dx = \int (3x^7 - 6x^3 + 3x^2)\, dx = 3\frac{x^8}{8} - 6\frac{x^4}{4} + 3\frac{x^3}{3} + C = \dots$

5) $\int (2x+1)\cdot(x-3)\, dx = \int (2x^2 - 6x + x - 3)\, dx = \int (2x^2 - 5x - 3)\, dx = 2\frac{x^3}{3} - 5\frac{x^2}{2} - 3x + c$

6) $\int (\sqrt{x} + \sqrt[3]{x}) \cdot x\, dx = \int \left(x^{\frac{1}{2}} \cdot x^1 + x^{\frac{1}{3}} \cdot x^1\right) dx = \int \left(x^{\frac{3}{2}} + x^{\frac{4}{3}}\right) dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^{\frac{7}{3}}}{\frac{7}{3}} + c = \dots$

$x^a \cdot x^b = x^{a+b}$

7) $\int \frac{2x^3 - 3x^2 + x}{x^2}\, dx = \int \left(2x - 3 + \frac{1}{x}\right) dx = 2\frac{x^2}{2} - 3x + \ln|x| + c$

8) $\int \frac{\sqrt{x} + \sqrt[3]{x}}{x}\, dx = \int \left(\frac{x^{\frac{1}{2}}}{x^1} + \frac{x^{\frac{1}{3}}}{x^1}\right) dx = \int \left(x^{-\frac{1}{2}} + x^{-\frac{2}{3}}\right) dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + c = \dots$

$x^{\frac{1}{2}-1}$ LEBO $\frac{x^a}{x^b} = x^{a-b}$

9) $\int \frac{x^2 + 4x + 3}{x+1}\, dx = \int \frac{\cancel{(x+1)}(x+3)}{\cancel{(x+1)}}\, dx = \int (x+3)\, dx = \frac{x^2}{2} + 3x + c$

10) $\int \frac{x^2 + 4x - 5}{x-1}\, dx = \int \frac{\cancel{(x-1)}(x+5)}{\cancel{x-1}}\, dx = \int (x+5)\, dx = \frac{x^2}{2} + 5x + c$

11) $\int (2x+3)^2\, dx = \int (4x^2 + 12x + 9)\, dx = 4\frac{x^3}{3} + 12\frac{x^2}{2} + 9x + c$

$(a+b)^2 = a^2 + 2ab + b^2$

12) $\int (x^2 - x + 1)^2\, dx = \int (x^4 - x^3 + x^2 - x^3 + x^2 - x + x^2 - x + 1)\, dx = \int (x^4 - 2x^3 + 3x^2 - 2x + 1)\, dx =$

$(x^2 - x + 1)(x^2 - x + 1)$

$= \frac{x^5}{5} - 2\frac{x^4}{4} + 3\frac{x^3}{3} - 2\frac{x^2}{2} + x + c$

13) $\int \left(\frac{1}{3}\cos x + \frac{2\sin x}{5} + \frac{3 \cdot 1}{\cos^2 x} - \frac{1 \cdot 1}{2\sin^2 x}\right) dx = \frac{1}{3}\sin x + \frac{2}{5}(-\cos x) + 3\operatorname{tg} x - \frac{1}{2}(-\operatorname{cotg} x) + c$

14) $\int \left(e^x + 4^x - \left(\frac{2}{3}\right)^x\right) dx = e^x + \frac{4^x}{\ln 4} - \frac{\left(\frac{2}{3}\right)^x}{\ln \frac{2}{3}} + c$

15) $\int \left(2^x \cdot 3^x + \frac{6^x}{2^x} - \frac{1}{3^x}\right) dx = \int \left(6^x + 3^x - \left(\frac{1}{3}\right)^x\right) dx = \frac{6^x}{\ln 6} + \frac{3^x}{\ln 3} - \frac{\left(\frac{1}{3}\right)^x}{\ln \frac{1}{3}} + c$

$a^x \cdot b^x = (a \cdot b)^x \qquad \frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$

16) $\int \frac{2^x + 4^x}{6^x}\, dx = \int \left(\left(\frac{1}{3}\right)^x + \left(\frac{2}{3}\right)^x\right) dx = \frac{\left(\frac{1}{3}\right)^x}{\ln \frac{1}{3}} + \frac{\left(\frac{2}{3}\right)^x}{\ln \frac{2}{3}} + c$

17) $\int \frac{\overset{f'}{2x}}{\underset{f}{x^2 - 4}}\, dx = \ln|x^2 - 4| + c$

18) $\frac{1}{3}\int \frac{\overset{f'}{3 \cdot x^2}}{\underset{f}{x^3 + 1}}\, dx = \frac{1}{3}\ln|x^3 + 1| + c$

19) $-\int \frac{\overset{f'}{-\cos x}}{\underset{f}{2 - \sin x}}\, dx = -\ln|2 - \sin x| + c$

$\ln a - \ln b = \ln \frac{a}{b}$

$\ln a + \ln b = \ln(a \cdot b)$

20) $\int (\operatorname{cotg} x + \operatorname{tg} x)\, dx = \int \left(\frac{\overset{f'}{\cos x}}{\underset{f}{\sin x}} - \frac{\overset{f'}{-\sin x}}{\underset{f}{\cos x}}\right) dx = \ln|\sin x| - \ln|\cos x| + c =$

$= \ln|\operatorname{tg} x| + c$