

EXPONENCIÁLNY RAST

PR 1



$$Q(0) = 60 \text{ mil.}$$

$$Q(5) = 90 \text{ mil.}$$

$$Q(15) = ?$$

$$e^0 = 1$$

$$Q(t) = Q_0 e^{kt}$$

$$Q(0) = Q_0 \cdot e^{k \cdot 0} = Q_0 = 60$$

$$Q(t) = 60 e^{kt}$$

$$Q(5) = 60 e^{k \cdot 5} = 90 \quad | : 60$$

$$e^{k \cdot 5} = \frac{3}{2} \quad | \ln()$$

$$\ln e^{k \cdot 5} = \ln \frac{3}{2}$$

$$5k \cdot \ln e = \ln \frac{3}{2}$$

$$5k = \ln \frac{3}{2} \quad | : 5$$

$$k = \frac{1}{5} \ln \frac{3}{2}$$

$$Q(t) = 60 \cdot e^{\left(\frac{1}{5} \ln \frac{3}{2}\right) \cdot t}$$

$$Q(15) = 60 e^{\left(\frac{1}{5} \ln \frac{3}{2}\right) \cdot 15} = 60 e^{3 \ln \frac{3}{2}} =$$

$$= 60 \cdot \ln \left(\frac{3}{2}\right)^3 = 60 \cdot \left(\frac{3}{2}\right)^3 =$$

$$= 202,5 \text{ miliónov}$$

$$\frac{90}{60} = \frac{3}{2}$$

$$\log_a X = n \cdot \log_a X$$

$$\ln e = 1$$

$$\left(\ln \frac{3}{2}\right) \cdot 3 = \ln \left(\frac{3}{2}\right)^3$$

$$f \circ f^{-1} = id$$

PR 2

$$Q(0) = 2000$$

$$Q\left(\frac{1}{3}\right) = 6000$$

$$Q(1) = ?$$

t ... hodiny

$$20 \text{ minút} \quad t = \frac{1}{3}$$

$$Q(t) = Q_0 e^{kt}$$

$$Q(0) = Q_0 \cdot e^{k \cdot 0} = Q_0 = 2000$$

$$Q(t) = 2000 e^{kt}$$

$$Q\left(\frac{1}{3}\right) = 2000 e^{k \cdot \frac{1}{3}} = 6000 \quad | : 2000$$

$$e^{\frac{1}{3} \cdot k} = 3 \quad | \ln()$$

$$\ln e^{\left(\frac{1}{3} k\right)} = \ln 3$$

$$\ln e = 1$$

$$\frac{1}{3} k = \ln 3 \quad | \cdot 3$$

$$k = 3 \ln 3$$

$$(X^m)^n = X^{m \cdot n} \quad \checkmark$$

$$Q(t) = 2000 \cdot e^{(3 \ln 3) \cdot t} = 2000 e^{(\ln 3^3) \cdot t} =$$

$$= 2000 \cdot (e^{\ln 27})^t = 2000 \cdot 27^t \quad 27 \text{ krátka}$$

$$Q(1) = 2000 \cdot 27^1 =$$

$$= 54000$$

EXPONENCIÁLNE KLESANIE

PR2 $Q(t) = Q_0 \cdot e^{-0,003t}$

$Q_0 \xrightarrow{t=?} \frac{Q_0}{2}$

$Q_0 e^{-0,003t} = \frac{Q_0}{2}$ (POČAS ZOBRAZU)

$e^{-0,003t} = \frac{1}{2} \quad | \ln() \quad \frac{1}{2} = 2^{-1}$

$\boxed{-0,003t} = \ln \frac{1}{2} = \ln 2^{-1} = \boxed{-1 \ln 2} \quad | \cdot (-1)$

$0,003t = \ln 2 \quad | \cdot \frac{1000}{3} \quad | : 0,003 = \frac{3}{1000}$

$t = \frac{1000}{3} \ln 2$

$0,003 = \frac{3}{1000}$

$t = 231,049 \text{ rokov}$

PR3

$Q_0 \xrightarrow{t=1690} \frac{Q_0}{2}$

$Q(1690) = \frac{Q_0}{2}$

$Q(t) = Q_0 e^{-kt}$

$Q_0 e^{-k \cdot 1690} = \frac{Q_0}{2} \quad | : Q_0$

$e^{-1690k} = \frac{1}{2} \quad | \ln() \quad \frac{1}{2} = 2^{-1}$

$-1690k = \ln \frac{1}{2} = -\ln 2 \quad | \cdot (-1)$

$1690k = \ln 2 \quad | : 1690$

$k = \frac{\ln 2}{1690} \doteq 0,0004$

$Q(t) = Q_0^{=50} e^{-0,0004t} \quad Q_0 = 50 \xrightarrow{t=?} 5$

$50 e^{-0,0004t} = 5 \quad | : 50$

$e^{-0,0004t} = \frac{1}{10} \quad | \ln() \quad \frac{1}{10} = 10^{-1}$

$-0,0004t = \ln \frac{1}{10} = -\ln 10 \quad | \cdot (-1)$

$0,0004t = \ln 10 \quad | : 0,0004 \Rightarrow | \cdot \frac{10000}{4}$

$t = \frac{\ln 10}{0,0004} \quad (= \frac{10000}{4} \cdot \ln 10)$

$t \doteq 5616,06 \text{ rokov}$

PR1 $Q(t) = Q_0 e^{-0,04t}$

$Q(20) = 8986,58\$$

$Q_0 = 2 \quad (= Q(0))$

$\frac{4}{100} \cdot 20 = \frac{4}{5} = \frac{8}{10}$

$$Q(20) = 8986,58\$$$

$$Q_0 = ? \quad (= Q(0))$$

$$Q(20) = Q_0 \cdot e^{-0,04 \cdot 20} = 8986,58$$

$$Q_0 \cdot e^{-0,8} = 8986,58 \quad /: e^{-0,8} \checkmark$$

$$Q_0 = \frac{8986,58}{e^{-0,8}} = \cdot e^{0,8} \checkmark$$

$$Q_0 = 8986,58 \cdot e^{0,8}$$

$$Q_0 = 20000 \$$$

KRIVKA UČENIASA

$$Q(t) = B - A e^{-kt}$$

PR1

$$Q(t) = 700 - 400 \cdot e^{-0,5t} = B - A e^{-kt}$$

$$1) Q(0) = ?$$

$$2) Q(6) = ?$$

$$3) \lim_{t \rightarrow \infty} Q(t) = ?$$

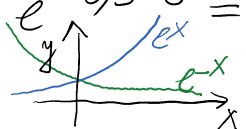
$$1) Q(0) = 700 - 400 \cdot e^{-0,5 \cdot 0} = 700 - 400 \cdot e^0 =$$

$$= 700 - 400 = (= B - A)$$

$$= 300$$

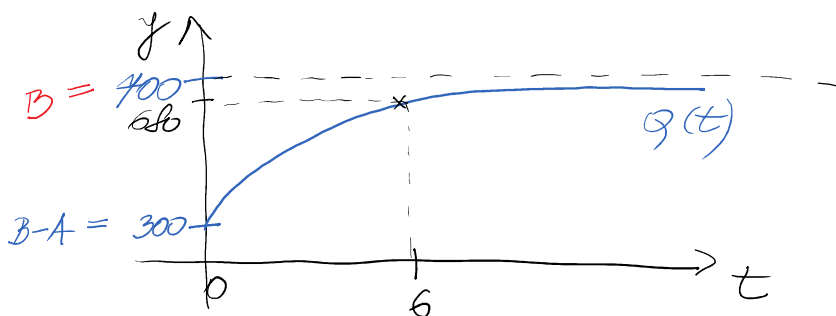
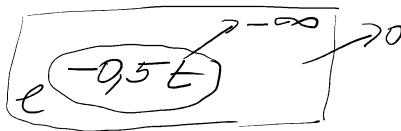
$$2) Q(6) = 700 - 400 \cdot e^{-0,5 \cdot 6} = 700 - 400 \cdot e^{-3} =$$

$$= 680$$



$$3) \lim_{t \rightarrow \infty} Q(t) = \lim_{t \rightarrow \infty} 700 - 400 \cdot e^{-0,5t}$$

$$= 700 - 400 \cdot 0 = 700 (= B)$$



PR2

$$Q(0) = 300$$

$$Q(6) = 410$$

$$Q(t) = 500 - A e^{-kt}$$

k, A ?

$$Q(0) = 500 - A \cdot e^{-k \cdot 0} = 300$$

$$500 - A = 300$$

$$A = 500 - 300 = 200$$

$$Q(t) = 500 - 200 e^{-kt} \quad \checkmark$$

$$Q(6) = 500 - 200 \cdot e^{-k \cdot 6} = 410 \quad | -410 + 200 \cdot e^{-6k}$$

$$200 e^{-6k} = 500 - 410$$

$$200 e^{-6k} = 90 \quad | :200$$

$$e^{-6k} = \frac{9}{20} \quad | \ln()$$

$$-6k = \ln \frac{9}{20} = -\ln \frac{20}{9}$$

$$6k = \ln \frac{20}{9} \quad | :6$$

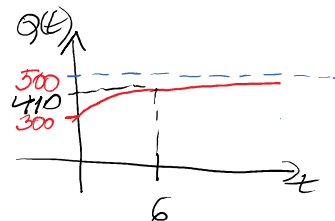
$$k = \frac{1}{6} \ln \frac{20}{9} \doteq 0,133$$

$$\ln e^{-6k} = -6k$$

$$\frac{9}{20} = \left(\frac{20}{9}\right)^{-1} \quad \checkmark \ln \left(\frac{20}{9}\right)^{-1}$$

$$Q(t) = 500 - 200 e^{-\left(\frac{1}{6} \ln \frac{20}{9}\right)t} =$$

$$= 500 - 200 e^{-0,133t}$$



LOGISTICKÁ KRIVKA

$$Q(t) = \frac{B}{1 + A e^{-Bkt}}$$

PR1

$$f(t) = \frac{10}{1 + 9 \cdot e^{-0,4t}}$$

1) $f(0) = ?$

2) $\lim_{t \rightarrow \infty} f(t) = ?$

3) graf

4) $f(3) = ?$

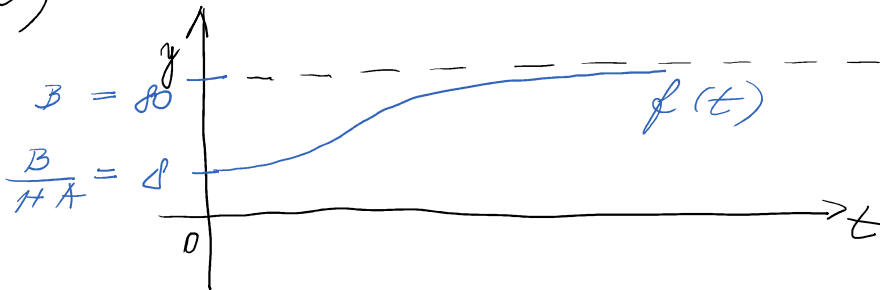
$f(5) = ?$

$f(7) = ?$

$$1) f(0) = \frac{80}{1+9} = \frac{80}{10} = 8 \quad (= \frac{B}{1+A})$$

$$2) \lim_{t \rightarrow \infty} \frac{80}{1+9 \cdot e^{-0.4t}} = \frac{80}{1} = 80 \quad (= B)$$

3)



$$4) f(3) = \frac{80}{1+9 \cdot e^{-0.4 \cdot 3}} = 22$$

$$f(5) = \frac{80}{1+9 \cdot e^{-0.4 \cdot 5}} = 36$$

$$f(7) = \frac{80}{1+9 \cdot e^{-0.4 \cdot 7}} = 52$$

$$\text{PR 2} \quad P(t) = \frac{20}{2+3e^{-0.06t}}$$

$$= \frac{20}{2(1 + \frac{3}{2}e^{-0.06t})} = \frac{10}{1 + \frac{3}{2}e^{-0.06t}} = \frac{10}{1+Ae^{-Bt}} \quad \checkmark$$

$$1) P(0) = ?$$

$$2) \lim_{t \rightarrow \infty} P(t) = ?$$

3) graf

$$4) P(15) = ?$$

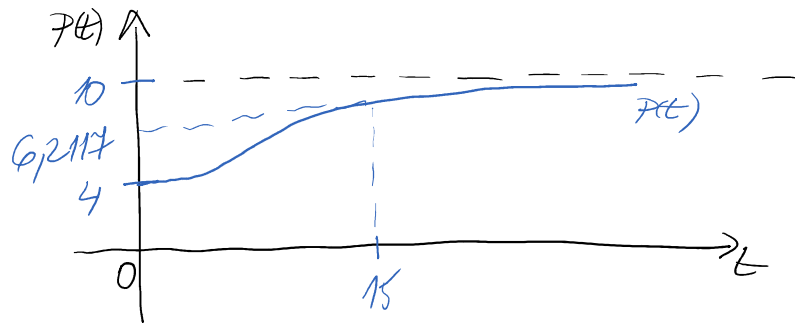
$$P(0) = \frac{10}{1+\frac{3}{2}} = \frac{10}{\frac{5}{2}} = \frac{20}{5} = 4$$

$$1) P(0) = \frac{20}{2+3} = 4 \text{ milioni}$$

$$2) \lim_{t \rightarrow \infty} \frac{20}{2+3 \underbrace{e^{-0.06t}}_{\rightarrow 0}} = \frac{20}{2+3 \cdot 0} = 10 \text{ mil.}$$

3) $P(t) \uparrow$

3)



$$4) P(15) = \frac{20}{2 + 3 e^{-0,06 \cdot 15}} = \underline{6,2117 \text{ millones}}$$

$$Q(t) = B (\quad)^{-1} = -1 \cdot B (\quad)^{-2} \cdot (\quad)'$$