OPTIMALIZÁCIA PRIEMERNÝCH NÁKLADOV (PRÍJMOV)

PR1 $\underline{C(q) = 3q^2 + q + 48}$

1) $AC(q)$ MINIMÁLNE
2) $AC(q) \stackrel{?}{=} MC(q)$
3) GRAFY $AC(q)$, $MC(q)$

1) $AC(q) = \dfrac{C(q)}{q} = \dfrac{3q^2+q+48}{q}$ $\quad D(f) = R - \{0\}$ $\quad \boxed{q>0} \Rightarrow D(f) = R^+ = (0, \infty)$

$$AC'(q) = \frac{(6q+1)\cdot q - (3q^2+q+48)\cdot 1}{q^2} =$$

$$= \frac{6q^2 + \cancel{q} - 3q^2 - \cancel{q} - 48}{q^2} =$$

$$= \frac{3q^2-48}{q^2}$$

S.B. $AC'(q) = 0$

$3q^2 - 48 = 0 \quad /\cdot\frac{1}{3}$ RESP. $3(q^2-16) = 0$ $\quad a^2-b^2=(a-b)(a+b)$

$q^2 = 16 \quad /\sqrt{\ }$ $\qquad (q-4)(q+4) = 0$

$q = \pm 4$

$\boxed{q>0}$ $\quad q = 4$ STACIONÁRNY BOD

URČENIE EXTRÉMU V STACIONÁRNOM BODE

1. SPÔSOB: $AC''(q) = \dfrac{6q\cdot q^2 - (3q^2-48)2q}{q^4}$

$$= \frac{\cancel{q}(6q^2 - 6q^2 + 96)}{q^{\cancel{4}\,3}} = \frac{96}{q^3}$$

$AC''(4) = \dfrac{96}{4^3} = \boxed{\dfrac{96}{64} > 0}$ $\Rightarrow$ LOKÁLNE MINIMUM V BODE $q = 4$

$AC(4) = 25$

II. SPÔSOB $\quad AC'(q) = \dfrac{3q^2-48}{q^2} = \dfrac{3(q^2-16)}{q^2} = \dfrac{3(q+4)(q-4)}{q^2}$

$AC'(q)$: $+ \quad - \quad - \quad +$ $\qquad x \qquad q>0$

$\qquad\qquad -4 \quad 0 \quad 4$

$AC(q)$: $\nearrow \quad \searrow \quad \searrow \quad \nearrow$

BOD LOKÁLNEHO MINIMA

FUNKCIA $AC(q)$ NADOBÚDA V BODE $q = 4$ LOKÁLNE MINIMUM $AC(4) = 25$

2) $AC(q) \stackrel{?}{=} MC(q)$

$\dfrac{C(q)}{q} = C'(q)$

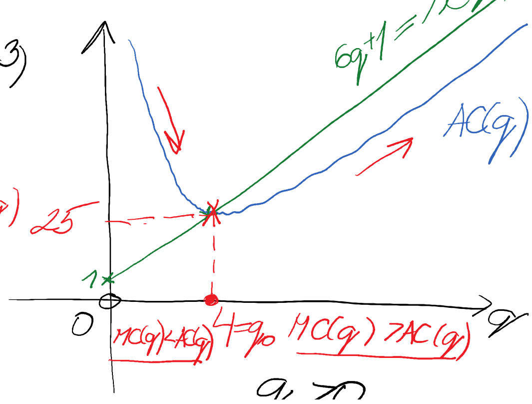
$AC(q) = \dfrac{3q^2+q+48}{q} = 6q + 1 = MC(q)$

$3q^2+q+48 = 6q^2+q$

$3q^2 = 48$

$q^2 = 16$

$q = \pm 4$

3) 

$$3q^2 = 48$$
$$q^2 = 16$$
$$q = \pm 4$$
$q > 0 \quad q = 4$

$0 \;|\; MC(q) < AC(q) \;|\; 4 = q_0 \;\; MC(q) > AC(q) \quad q$

$q > 0$

PR2 $\quad R(q) = -2q^2 + 68q - 128$

1) $AR(q) \overset{?}{=} MR(q)$
2) MONOTÓNNOSŤ $AR(q)$
3) GRAFY $AR(q), MR(q)$

$$AR(q) = \frac{R(q)}{q} = \frac{-2q^2+68q-128}{q} \qquad D(f) = (0, \infty)$$

$$MR(q) = R'(q) = -4q + 68$$

$$AR(q) = MR(q)$$

$$\frac{-2q^2+68q-128}{q} = -4q+68 \;\; (= MR(q)) \quad / \cdot q$$

$$-2q^2 + \cancel{68q} - 128 = -4q^2 + \cancel{68q} \quad / +4q^2$$

$$2q^2 - 128 = 0 \quad / : 2$$

$$q^2 = 64 \qquad q^2 - 64 = 0$$

$$q = \pm 8 \qquad (q-8)(q+8) = 0$$

$q > 0 \qquad \underline{q = 8}$

2) $$AR(q) = \frac{-2q^2+68q-128}{q}$$

$$AR'(q) = \frac{(-4q+68)q - (-2q^2+68q-128)\cdot 1}{q^2} =$$

$$= \frac{-4q^2 + \cancel{68q} + 2q^2 - \cancel{68q} + 128}{q^2} =$$

$$= \frac{-2q^2+128}{q^2} = \frac{-2(q^2-64)}{q^2} = \frac{-2(q+8)(q-8)}{q^2}$$

S.B. $\quad AR'(q) = 0$

$$-2q^2 + 128 = 0$$
$$q^2 = 64$$
$q > 0 \qquad q = \pm 8$
$$q = 8$$

$AR'(q) \quad 0 \quad (+) \quad 8 \quad (-) \quad q$

$AR(q) \quad 0 \quad \nearrow \quad 8 \quad \searrow$

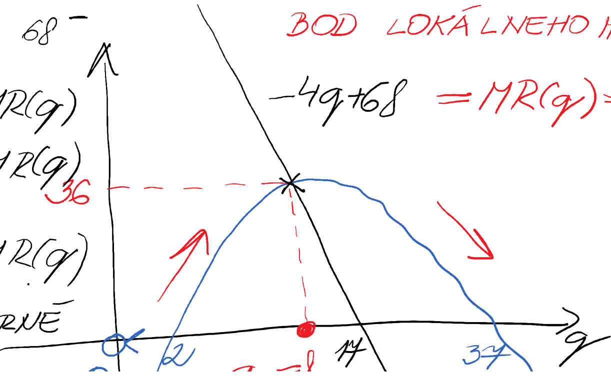
BOD LOKÁLNEHO MAXIMA

$0 < q < 8 \qquad AR(q) < MR(q)$

$8 < q < \infty \;(37) \qquad AR(q) > MR(q)$

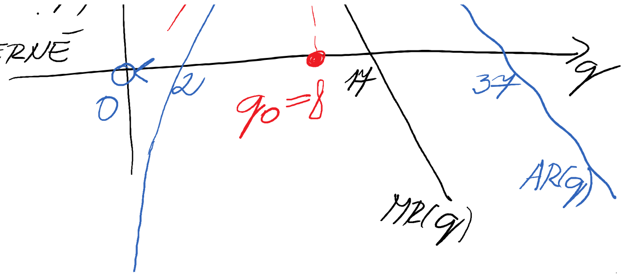
$q = 8 \qquad AR(q) = MR(q)$

MAXIMÁLNE PRIEMERNÉ PRÍJMY



MAXIMÁLNE PRIEMERNÉ PRÍJMY

$AR(8)=36$



OPTIMALIZÁCIA CELKOVÝCH VELIČÍN

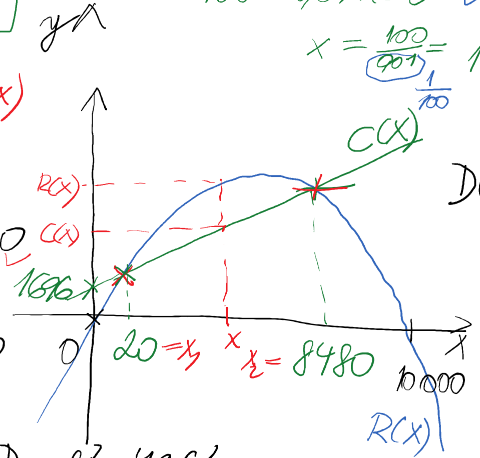
PR 1 $C(x) = 15x + 1696$

$R(x) = 100x - 0{,}01x^2 = \boxed{x(100-0{,}01x)}$

$100 - 0{,}01x = 0$ ✓

$x = \frac{100}{0{,}01} = 10\,000$, $\quad 0{,}01 = \frac{1}{100}$

1) GRAFY
2) ZISKOVÁ VÝROBA $R(x) > C(x)$
3) MAXIMÁLNY ZISK



$D(R) = \langle 0, 10\,000 \rangle$

1) $C(x) = R(x)$ ✓ $\qquad \underbrace{R(x) - C(x)}_{=P(x)} = 0$

$15x + 1696 = 100x - 0{,}01x^2$

$0{,}01x^2 - 85x + 1696 = 0 \quad / \cdot 100$

$x^2 - 8500x + 169\,600 = 0$

$(x-20)(x-8480) = 0 \qquad D = b^2 - 4ac$

$x_1 = 20 \qquad x_2 = 8480$

$\vdots \quad x_1 = 20,\ x_2 = 8480$

$R(x_1) = C(x_1)$

$R(x_2) = C(x_2)$

2) $R(x) > C(x)$

$x \in (20, 8480)$ VÝROBA ZISKOVÁ

$x \in \langle \underset{=a}{20}, \underset{=b}{8480} \rangle$ NA TOMTO INTERVALE HĽADÁME ABSOLÚTNE MAXIMUM

$\langle a, b \rangle = \langle 20, 8480 \rangle$

3) $P(x) = R(x) - C(x) =$

$= 100x - 0{,}01x^2 - (15x + 1696)$

$= 100x - 0{,}01x^2 - 15x - 1696$

$= -0{,}01x^2 + 85x - 1696$ ✓ $\quad \cancel{/ \cdot (-0{,}01)}$!

$P'(x) = -0{,}02x + 85$ ✓

S.B. $P'(x) = 0$

$-0{,}02x + 85 = 0$

$0{,}02x = 85 \quad / \cdot \frac{100}{2} \qquad \frac{2}{100} = 0{,}02$

$x = 4250 \in \langle 20, 8480 \rangle$

$\max_{\langle a,b \rangle} f(x) = \{f(x_1), f(x_2), \ldots, f(x_n), f(a), f(b)\}$

$P''(x) = -0{,}02$

$P''(4250) = -0{,}02 < 0$ LOKÁLNE MAXIMUM V $x = 4250$

$P(4250) = 178\,929$

$$P(4250) = 178\ 929$$

GLOBÁLNE MAXIMUM NA $\langle a,b\rangle = \langle 20, 8480\rangle$

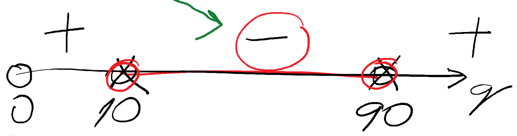
$$\max_{\langle 20,8480\rangle} P(x) = \max\{P(4250), \underbrace{P(20)}_{=0}, \underbrace{P(8480)}_{=0}\} = \max\{178\ 929, 0, 0\} = 178\ 929$$

ÚROVEŇ PRODUKCIE, PRI KTOREJ FIRMA DOSAHUJE MAXIMÁLNY ZISK 178 929, JE 4250 KUSOV.

PR3.
$K = 360$ náklad 1ks
$V(q) = (0{,}2q + 10)\cdot q$
$p = 50 - 0{,}2q$ 1ks

1) $R(q) > C(q)$
2) $R(q)$ MAXIMÁLNE
3) $P(q)$ MAXIMÁLNE

1) $C(q) = V(q) + K = (0{,}2q+10)q + 360 = 0{,}2q^2 + 10q + 360$
$R(q) = p\cdot q = (50 - 0{,}2q)q = 50q - 0{,}2q^2$
$R(q) > C(q)$
$50q - 0{,}2q^2 > 0{,}2q^2 + 10q + 360$
$0 > 0{,}4q^2 - 40q + 360 \quad / \cdot \frac{10}{4}$
$q^2 - 100q + 900 \boxed{< 0}$
$(q-10)(q-90) < 0$
$q_1 = 10 \qquad q_2 = 90$



ziskový $q \in (10, 90)$

2) $R(q) = 50q - 0{,}2q^2$
$R'(q) = 50 - 0{,}4q$
S.B. $R'(q) = 0$
$50 - 0{,}4q = 0$
$0{,}4q = 50 \quad / \cdot \frac{10}{4} \; (:0{,}4)$ $\quad C(q) > R(q)$
$q = 125 \notin (10, 90)$ (NIE JE ZISKOVÁ)

$R''(q) = -0{,}4 < 0$ LOKÁLNE MAXIMUM V BODE $q = 125$

PRÍJMY SÚ MAXIMÁLNE PRE $q = 125$ KUSOV
$R(125) = 3125$

3) $P(q) = R(q) - C(q) = -0{,}4q^2 + 40q - 360$
$P'(q) = -0{,}8q + 40$
S.B. $P'(q) = 0$
$-0{,}8q + 40 = 0$
$0{,}8q = 40 \quad / \cdot \frac{10}{8}$

$q = 50 \in (10, 90)$ ✓

$P''(q) = -0{,}8 < 0$ LOKÁLNE MAXIMUM

ZISK JE MAXIMÁLNY PRI POČTE $q = 50$ ks $P(50) = 640$
(LOKÁLNY AJ GLOBÁLNY EXTRÉM NA $\langle 10, 90 \rangle$)

PR4

	STARÉ	NOVÉ
R:	$p = 6$ ✓, $q = 3\,000$ ✓	$\boxed{p = 6 + x \cdot 1,\ q = 3000 - x \cdot 1000}$
C:	$p = 4$ ✓, $q = 3000$	$p = 4$, $q = 3000 - 1000x$

✓ x ... POČET ZVÝŠENÍ CENY O 1\$

$R(x) = p \cdot q = (6+x)(3000 - 1000x)$

$C(x) = p \cdot q = 4(3000 - 1000x)$

P: $p = (6+x) - 4 = 2 + x$

$q = 3000 - 1000x$

$P(x) = R(x) - C(x)$

$= (2+x)(3000 - 1000x)$

$P(x)$ MAXIMÁLNE

$R(x) = p \cdot q = (6+x)(3000 - 1000x)$

$C(x) = 4(3000 - 1000x)$

$P(x) = R(x) - C(x) =$

$= (6+x)(3000 - 1000x) - 4(3000 - 1000x) =$

$= (6 + x - 4)(3000 - 1000x) = \underbrace{(2+x)}_{\text{ZISK Z 1 KS}}(3000 - 1000x) =$

$= 1000\,(2+x) \cdot (3-x)$

$P'(x) = 1000\,[1 \cdot (3-x) + (2+x) \cdot (-1)] =$

$= 1000\,(3 - x - 2 - x) = 1000\,(1 - 2x)$

S.B. $P'(x) = 0$

$1000(1 - 2x) = 0$

$1 - 2x = 0$

$x = \frac{1}{2}$

$P''(x) = 1000 \cdot (-2) = -2000 < 0$ LOKÁLNE MAXIMUM

$p = 6 + x = 6 + \frac{1}{2} = 6{,}5$ \$

PRI CENE $p = 6{,}5$ \$ ZA KUS DOSIAHNE MAXIMÁLNY ZISK
$P(6{,}5) = 6250$

PR5

	STARÉ	NOVÉ
U:	$q = 60$	$q = 60 + x$
	$p = 400$ ↑ÚRODA 1 STROMU	$p = 400 - 4x$

x ... POČET DODATOČNE ZASADENÝCH STROMOV

$U(x)$ MAXIMÁLNA ÚRODA

$U(x)$ MAXIMÁLNA ÚRODA

$U(x) = p \cdot q = (60+x)\cdot(400-4x)$

$U'(x) = 1\cdot(400-4x) + (60+x)\cdot(-4) =$

$= 400-4x-240-4x =$

$= 160-8x$

S.B. $U'(x) = 0$

$160-8x = 0$

$8x = 160$

$x = 20$

! $U''(x) = -8 < 0$ LOKÁLNE MAXIMUM

ÚRODA JE MAXIMÁLNA, AK DODATOČNE ZASADÍ 20 STROMOV $U(20) = 25\,600$

PR 2)

$C(q) = 5q$

$p = 25-2q$ (PREDAJNÁ CENA)

1) $P(q)$ MAXIMÁLNY

2) $P(q)$ S DAŇOU t DOLÁROV ZA KUS

3) PRI q_0: $P(q_0)$ JE MAXIMÁLNE

MAXIMALIZOVAŤ VÝBER DANE

1) $R(q) = p\cdot q = (25-2q)\cdot q = 25q-2q^2$

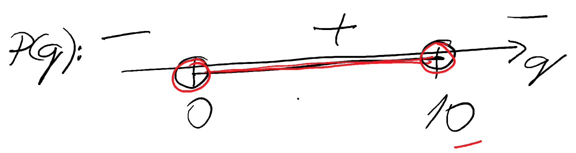
$C(q) = 5q$

$P(q) = R(q) - C(q) = 25q-2q^2-5q = 20q-2q^2$

$20q-2q^2 > 0$

$10q-q^2 > 0$

$q(10-q) > 0$



(0,10) ZISKOVÁ

$P'(q) = 20-4q$

S.B. $P'(q) = 0$

$20-4q = 0$

$q = 5 \in (0,10)$

$P''(q) = -4 < 0$ LOKÁLNE MAXIMUM

ZISK SPOLOČNOSTI JE MAXIMÁLNY PRI POČTE $q = 5$ KUSOV $P(5) = 50$

2) $f(t) = t\cdot q$ (DAŇ)

$P(q) = 20q-2q^2-t\cdot q$

$P'(q) = 20-4q-t$ PARAMETER

S.B. $P'(q) = 0$

$$20-4q-t=0$$
$$4q=20-t$$
$$q=5-\frac{t}{4} \geq 0 \qquad \boxed{t \leq 20}$$

$$P''(t) = -4 < 0 \quad \text{LOKÁLNE MAXIMUM}$$

PRI OBJEME PRODUKCIE $q=5-\frac{t}{4}$ KUSOV JE ZISK SPOLOČNOSTI MAXIMÁLNY, AK JE KAŽDÝ KUS ZDANENÝ SUMOU t DOLÁROV.

3) $$f(t) = t \cdot q = t\left(5-\frac{t}{4}\right) = 5t - \frac{1}{4}t^2$$
$$f'(t) = 5 - \frac{1}{4}\cdot 2t = 5 - \frac{1}{2}t$$

S.B. $$f'(t) = 0$$
$$5-\frac{1}{2}t = 0$$
$$\frac{1}{2}t = 5$$
$$t = 10 \checkmark \leq 20$$

$$f''(t) = -\frac{1}{2} < 0 \quad \text{LOKÁLNE MAXIMUM}$$

PRÍJEM Z DANE JE MAXIMÁLNY PRI DANI 10 \$ ZA KUS $\quad f(10)=25$