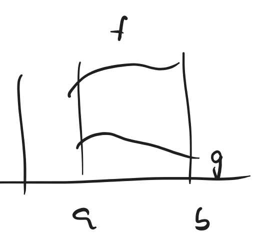
DERIVÁCIA V SMERE VEKTORA $\vec{u}$

(ZOVŠEOBECNENIE PARCIÁLNYCH DERIVÁCIÍ.)

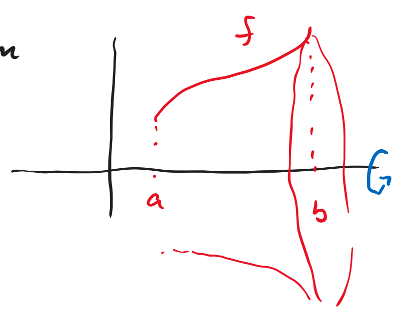
OTÁZKY: APLIKÁCIE $\int_a^b f(x)\,dx$

1° OS $A: a \le x \le b$, $g(x) \le y \le h(x)$

$P(A) = \int_a^b f(x) - g(x)\,dx$

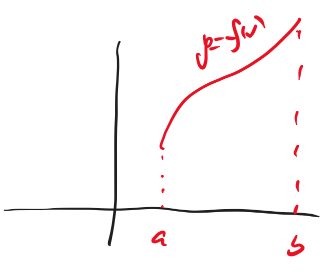


2° Objem



$V = \pi \int_a^b f^2(x)\,dx$

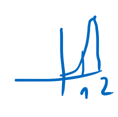
3° dĺžka krivky



$l = \int_a^b \sqrt{1 + (f'(x))^2}\,dx$

$f' = 2x$

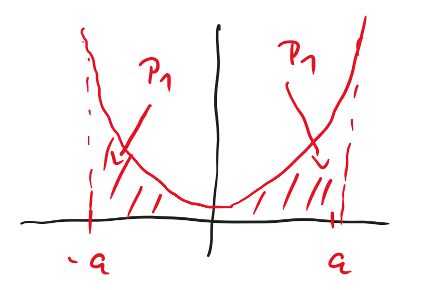
Nech $f(x) = x^2$ a sme na $\langle 1,2 \rangle$



$l = \int_1^2 \sqrt{1 + 4x^2}\,dx$

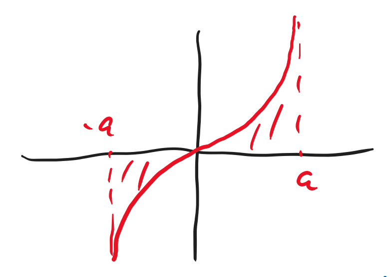
PÁRNE/NEPÁRNE FCIE

1° f je párna na $\langle -a, a \rangle$



$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

2. f - nepárna

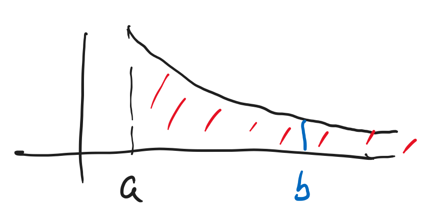


$\int_{-a}^{a} f(x)\,dx = 0$

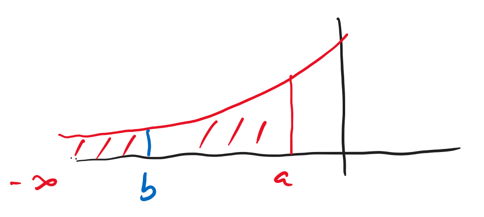
Priemerná hodnota fcie f na $\langle a,b \rangle$

$$\boxed{\text{priem. hod} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

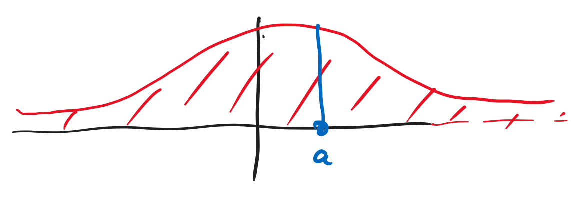
Nevl. int.



$\int_a^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx$



$\int_{-\infty}^{a} f(x)\,dx = \lim_{b\to-\infty} \int_b^a f(x)\,dx$



$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{a} f(x)\,dx + \int_a^{\infty} f(x)\,dx$

$a \in R$

DERIVÁCIA V SMERE $\vec{u}$

Samotné $\frac{\partial f(A)}{\partial x}$ (aj $\frac{\partial f(A)}{\partial y}$) v sebe obsahujú vektor (t.j. smer)

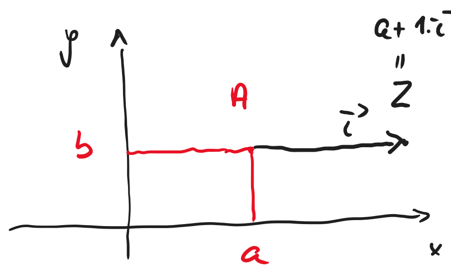
ukážeme na sublikácii

$A = [a,b]$

STARÁ, NOVÁ

subst. $x - a = t$, ak $x \to a$, tak $t \to 0$

$$\frac{\partial f(a,b)}{\partial x} = \lim_{x\to a} \frac{f(x,b) - f(a,b)}{x - a} = \lim_{t\to 0} \frac{f(a+t,b) - f(a,b)}{t}$$

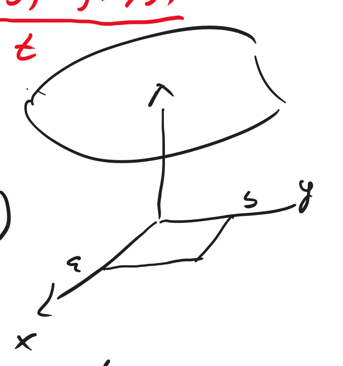


$a + 1 \cdot \vec{i}$

$[a+t, b] =$

$(a,b) + (t,0) = (a,b) + t(1,0)$

$(a,b) + t\vec{i} = A + t\vec{i}$



$$\frac{\partial f(A)}{\partial x} = \lim_{t\to 0} \frac{f(A + t\vec{i}) - f(A)}{t}$$

$\frac{\partial f(A)}{\partial x}$ hovorí o správaní sa fcie f v smere jednotkového vektora $\vec{i}$

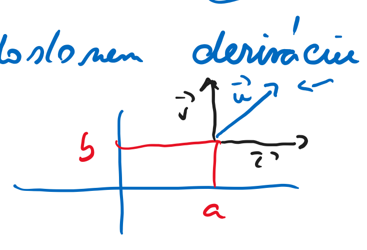
$$\frac{\partial f(A)}{\partial y} = \lim_{t\to 0} \frac{f(A + t\vec{j}) - f(A)}{t}$$

$\vec{j} = (0,1)$

aby nejaký smery mali rovnaké podm.

Ak vektor $\vec{i}$ nahradím všeobecným vektorom $\vec{u}$ (jednotkovým!) |u|=1

dostanem deriváciu funkcie v smere vektora $\vec{u}$



t.j. $$\boxed{\frac{df(A)}{d\vec{u}} = \lim_{t\to 0} \frac{f(A + t\vec{u}) - f(A)}{t}}$$ OT2

$|\vec{u}| = 1$ len.

v špec. prípade ak $\vec{u} = \vec{i}$ $\frac{df}{d\vec{i}} = \frac{\partial f(A)}{\partial x}$ ak $\vec{u} = \vec{j} \Rightarrow \frac{df}{d\vec{j}} = \frac{\partial f(A)}{\partial y}$

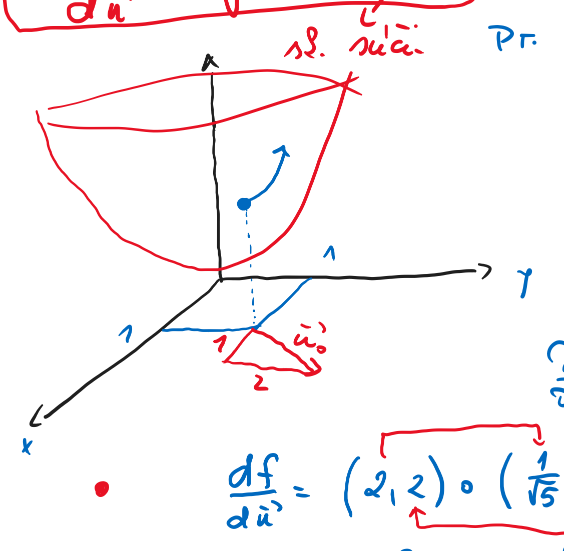
1. VÝPOČET $\frac{df}{d\vec{u}}$

2. POUŽITIE

GRADIENT $f(x,y)$ v A je vektor $\overrightarrow{\text{grad} f(A)} = \left(\frac{\partial f(A)}{\partial x}, \frac{\partial f(A)}{\partial y}\right)$ OT 1

$$\boxed{\frac{df(A)}{d\vec{u}} = \overrightarrow{\text{grad} f(A)} \circ \vec{u}} = \frac{\partial f(A)}{\partial x}u_1 + \frac{\partial f(A)}{\partial y}u_2$$ OT 3

sk. súčin



Pr.

$z = f(x,y) = x^2 + y^2$

$A = [1,1] \Rightarrow f(A) = 2$

$\vec{u}_0 = (1,2)$ $|\vec{u}_0| = \sqrt{1+4} = \sqrt{5}$

$\vec{u} = \frac{1}{\sqrt{5}}(1,2) = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$

$\frac{\partial f}{\partial x} = 2x \Rightarrow \frac{\partial f(A)}{\partial x} = 2 = \frac{\partial f(A)}{\partial y}$

- $\frac{df}{d\vec{u}} = (2,2) \circ \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right) = \frac{2}{\sqrt{5}} + \frac{4}{\sqrt{5}} = \frac{6}{\sqrt{5}} > 0$ t.j. f v smere $\vec{u}$ ↑

V ktorom smere funkcia najviac rastie??? (v opačnom smere najviac klesá)

najväčší je pre $\vec{u}_0 = \overrightarrow{\text{grad} f(A)} = (2,2)$ $|\vec{u}_0| = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

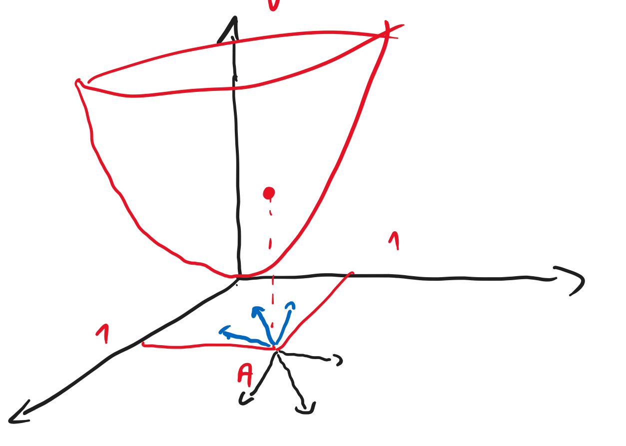
$\frac{36}{5} < 8$

- $\vec{u} = \frac{1}{2\sqrt{2}}(2,2) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\frac{df(A)}{\overrightarrow{\text{grad} f(A)}} = (2,2) \circ \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{4}{\sqrt{2}}$ → $\frac{16}{2} = 8$ ← NAJVÄČŠÍ RAST

sk. súčin

$\frac{df(A)}{-\overrightarrow{\text{grad} f(A)}} = -\frac{4}{\sqrt{2}}$ najväčší pokles



$\text{grad } f(A) = (2,2)$

$-\text{grad } f(A) = (-2,-2)$ smer z A do [0,0] kde je lok. min.