$z = \underline{f(x,y)}$ ak $\boxed{g(x,y)=0}$

$L(x,y) = f(x,y) + \lambda \cdot g(x,y) \quad \lambda \in \mathbb{R}$

---

Pr: $z = 8-2x-4y \; {=}f(x)$ ak $\boxed{x^2+2y^2=12} \Rightarrow \boxed{x^2+2y^2-12=0}$ $\to g(x,y)$

$L(x,y) = 8-2x-4y+\lambda(x^2+2y^2-12) \quad \lambda \in \mathbb{R}$

der. 1. rádu

$\boxed{\begin{aligned} L'_x &= -2+2\lambda x \\ L'_y &= -4+4\lambda y \end{aligned}}$

$-2+2\lambda x = 0 \to x = \frac{1}{\lambda}$

$-4+4\lambda y = 0 \to y = \frac{1}{\lambda}$

$\boxed{x^2+2y^2=12}$

$\frac{1}{\lambda^2} + \frac{2}{\lambda^2} = 12$

$\frac{3}{\lambda^2} = 12 \quad /\cdot \lambda^2$

$3 = 12\lambda^2 \quad /:12$

$\frac{1}{4} = \lambda^2 \quad /\sqrt{}$

$\frac{1}{2} = |\lambda|$

$\pm\frac{1}{2} = \lambda$

$\sqrt{x^2} \neq x \quad$ ✗

$\sqrt{x^2} = |x| \quad$ ○

| $\lambda = \frac{1}{2}$ | $\lambda = -\frac{1}{2}$ |
|---|---|
| $x = 2$ | $x = -2$ |
| $y = 2$ | $y = -2$ |
| $A[2;2]$ | $B[-2;-2]$ |

SB

| der. 2. rádu: | $\lambda=\frac{1}{2}; A[2,2]$ | $\lambda=-\frac{1}{2}\; B[-2;-2]$ |
|---|---|---|
| $L''_{xx} = 2\lambda$ | $\boxed{1} > 0$ | $\boxed{-1} < 0$ |
| $L''_{xy} = 0$ | 0 | 0 |
| $L''_{yx} = 0$ | 0 | 0 |
| $L''_{yy} = 4\lambda$ | 2 | $-2$ |

$D_2(A) = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2 > 0$

$D_2(B) = \begin{vmatrix} -1 & 0 \\ 0 & -2 \end{vmatrix} = 2 > 0$

$f'' > 0$ ∪

$f'' < 0$ ∩

v A má fcia z viaz. lok. min.

v B má fcia z viaz. lok. max

---

Pr: $z = -3+2x+4y \; {=}f(x,y)$ ak $\boxed{2x^2+y^2=\frac{1}{2}} \Rightarrow 2x^2+y^2-\frac{1}{2}=0$ $\to g(x,y)$

$L(x,y) = -3+2x+4y+\lambda\left(2x^2+y^2-\frac{1}{2}\right) = 0$

der. 1. rádu

$L'_x = 2+4\lambda x$

$L'_y = 4+2\lambda y$

$2+4\lambda x = 0 \to x = -\frac{1}{2\lambda}$

$4+2\lambda y = 0 \to y = -\frac{2}{\lambda}$

$2x^2+y^2 = \frac{1}{2}$

$2\left(-\frac{1}{2\lambda}\right)^2 + \left(-\frac{2}{\lambda}\right)^2 = \frac{1}{2}$

$\frac{1}{2\lambda^2} + \frac{4}{\lambda^2} = \frac{1}{2} \quad /\cdot 2\lambda^2$

$1+8 = \lambda^2$

$9 = \lambda^2 \quad /\sqrt{}$

$3 = |\lambda|$

$\pm 3 = \lambda$

$\lambda = 3$

$x = -\frac{1}{2\cdot 3} = -\frac{1}{6}$

$y = -\frac{2}{3} = -\frac{2}{3}$

$\lambda = -3$

$x = -\frac{1}{2(-3)} = -\frac{1}{-6} = \frac{1}{6}$

$y = -\frac{2}{-3} = \frac{2}{3}$

| $\lambda = 3$ | $\lambda = -3$ |
|---|---|
| $x = -\frac{1}{6}$ | $x = \frac{1}{6}$ |
| $y = -\frac{2}{3}$ | $y = \frac{2}{3}$ |
| $A\left[-\frac{1}{6}; -\frac{2}{3}\right]$ | $B\left[\frac{1}{6}; \frac{2}{3}\right]$ |

SB

| der. 2. rádu: | $\lambda=3\; A\left[-\frac{1}{6}; -\frac{2}{3}\right]$ | $\lambda=-3\; B\left[\frac{1}{6}; \frac{2}{3}\right]$ |
|---|---|---|
| $L''_{xx} = 4\lambda$ | $\boxed{12} > 0$ | $\boxed{-12} < 0$ |
| $L''_{xy} = 0$ | 0 | 0 |
| $L''_{yx} = 0$ | 0 | 0 |
| $L''_{yy} = 2\lambda$ | 6 | $-6$ |

$D_2(A) = \begin{vmatrix} 12 & 0 \\ 0 & 6 \end{vmatrix} > 0$

$D_2(B) = \begin{vmatrix} -12 & 0 \\ 0 & -6 \end{vmatrix} > 0$

v bode A má fcia z viaz. lok. min.

v bode B má fcia z viaz. lok. max

---

Pr: $z = \frac{1}{x} + \frac{1}{y} \; {=}f(x,y)$ ak $\boxed{\frac{1}{x^2}+\frac{1}{y^2}=2} \Rightarrow \frac{1}{x^2}+\frac{1}{y^2}-2=0$ $\to g(x,y)$

$L(x,y) = \frac{1}{x} + \frac{1}{y} + \lambda\left(\frac{1}{x^2}+\frac{1}{y^2}-2\right)$

der. 1. rádu:

$\boxed{\begin{aligned} L'_x &= -\frac{1}{x^2} - 2\lambda\frac{1}{x^3} \\ L'_y &= -\frac{1}{y^2} - 2\lambda\frac{1}{y^3} \end{aligned}}$

$-\frac{1}{x^2} - 2\lambda\frac{1}{x^3} = 0 \quad /\cdot(-x^3)$

$-\frac{1}{y^2} - 2\lambda\frac{1}{y^3} = 0 \quad /\cdot(-y^3)$

$\frac{1}{x^2} + \frac{1}{y^2} - 2 = 0$

$x + 2\lambda = 0 \to x = -2\lambda$

$y + 2\lambda = 0 \to y = -2\lambda$

$\frac{1}{x^2} + \frac{1}{y^2} = 2$

$\frac{1}{4\lambda^2} + \frac{1}{4\lambda^2} = 2$

$\frac{2}{4\lambda^2} = 2 \quad /\cdot\lambda^2$

$\frac{1}{2} = 2\lambda^2 \quad /:2$

$\frac{1}{4} = \lambda^2 \quad /\sqrt{}$

$\frac{1}{2} = |\lambda|$

$\pm\frac{1}{2} = \lambda$

| $\lambda = \frac{1}{2}$ | $\lambda = -\frac{1}{2}$ |
|---|---|
| $x = -1$ | $x = 1$ |
| $y = -1$ | $y = 1$ |
| $A[-1;-1]$ | $B[1;1]$ |

SB

| der. 2. rádu: | $\lambda=\frac{1}{2}\; A[-1,-1]$ | $\lambda=-\frac{1}{2}\; B[1;1]$ |
|---|---|---|
| $L''_{xx} = \frac{2}{x^3} + \frac{6\lambda}{x^4}$ | $\frac{2}{(-1)^3} + \frac{6\cdot\frac{1}{2}}{(-1)^4} = -2+3 = \boxed{1} > 0$ | $\frac{2}{1^3} + \frac{6\cdot(-\frac{1}{2})}{1^4} = 2-3 = \boxed{-1} < 0$ |
| $L''_{xy} = 0$ | 0 | 0 |
| $L''_{yx} = 0$ | 0 | 0 |
| $L''_{yy} = \frac{2}{y^3} + \frac{6\lambda}{y^4}$ | 1 | $-1$ |

$D_2(A) = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} > 0$

$D_2(B) = \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} > 0$

v bode A má fcia z viaz. lok. min

v bode B má fcia z viaz. lok. max.