P3: $z = 2x^3 + y^2 + 3x^2 - 3y - 12x + 1$

1. der. podle x: $z'_x = 6x^2 + 6x - 12$
2. der. podle y: $z'_y = 3y^2 - 3$

$6x^2 + 6x - 12 = 0 \quad /:6$
$3y^2 - 3 = 0 \quad /:3$

$x^2 + x - 2 = 0$
$(x+2)(x-1) = 0$
$x_1 = -2 \;;\; x_2 = 1$

$y^2 - 1 = 0$
$y^2 = 1 \quad /\sqrt{\;}$
$|y| = 1$
$y = \pm 1$

SB: $A[1;1]$
$B[1;-1]$
$C[-2;1]$
$D[-2;-1]$

der. 2. řádu

	$A[1;1]$	$B[1;-1]$	$C[-2;1]$	$D[-2;-1]$
$z''_{yx} = 12x + 6$	18	18	-18	-18
$z''_{xy} = 0$	0	0	0	0
$z''_{yx} = 0$	0	0	0	0
$z''_{yy} = 6y$	6	-6	6	-6
	>0 ∪			<0 ∩

A: $\Delta_2(A) = \begin{vmatrix} 18 & 0 \\ 0 & 6 \end{vmatrix} = 18 \cdot 6 - 0 > 0$
↑ MÁ EXTRÉM

B: $\Delta_2(B) = \begin{vmatrix} 18 & 0 \\ 0 & -6 \end{vmatrix} = 18 \cdot (-6) - 0 < 0$
↑ B je sedlový bod

↑ A má lok. minimum

C: $\Delta_2(C) = \begin{vmatrix} -18 & 0 \\ 0 & 6 \end{vmatrix} = (-18)\,6 - 0 < 0$
↑ C je sedlový bod

D: $\Delta_2(D) = \begin{vmatrix} -18 & 0 \\ 0 & -6 \end{vmatrix} = (-18)(-6) - 0 > 0$
↑ D MÁ EXTRÉM
↑ D má lok. max.

P2: $z = y^3 + 3x^2 y - 18x - 30y + 4 \qquad x, y \in \mathbb{R}$

der. 1 řádu:
$z'_x = 6xy - 18 \rightarrow 6xy - 18 = 0 \quad /:6$
$z'_y = 3y^2 + 3x^2 - 30 \rightarrow 3y^2 + 3x^2 - 30 = 0 \quad /:3$

$xy - 3 = 0 \Rightarrow x = \frac{3}{y}$

$y^2 + x^2 - 10 = 0$
$y^2 + \left(\frac{3}{y}\right)^2 - 10 = 0$
$y^2 + \frac{9}{y^2} - 10 = 0$
$y^4 - 10y^2 + 9 = 0$

substituce:
$t = y^2$
$t^2 - 10t + 9 = 0$
$(t-9)(t-1) = 0$
$t_1 = 9 \quad t_2 = 1$
$y = y^2/r \quad 1 = y^2 \; /r$
$9 = |y| \quad 1 = |y|$
$\pm 3 = y \quad \pm 1 = y$
$y_1 = 3 \quad y_2 = 1$
$y_3 = -3 \quad y_4 = -1$

SB: $A[3;1]$
$B[-3;-1]$
$C[1;3]$
$D[-1;-3]$

	$A[3;1]$	$B[-3;-1]$	$C[1;3]$	$D[-1;-3]$
$z''_{xx} = 6y$	6	-6	18	-18
$z''_{xy} = 6x$	18	-18	6	-6
$z''_{yx} = 6x$	18	-18	6	-6
$z''_{yy} = 6y$	6	-6	18	-18
			>0	<0

A: $\Delta_2(A) = \begin{vmatrix} 6 & 18 \\ 18 & 6 \end{vmatrix} = 6^2 - 18^2 < 0$
↑ MÁ sedlový bod

B: $\Delta_2(B) = \begin{vmatrix} -6 & -18 \\ -18 & -6 \end{vmatrix} = (-6)^2 - (-18)^2 < 0$
↑ B má sedlový bod

C: $\Delta_2(C) = \begin{vmatrix} 18 & 6 \\ 6 & 18 \end{vmatrix} = 18^2 - 6^2 > 0$
↑ C má extrém
↑ C má lok. minimum

D: $\Delta_2(D) = \begin{vmatrix} -18 & -6 \\ -6 & -18 \end{vmatrix} = (-18)^2 - (-6)^2 > 0$
↑ D má extrém
↑ D má lok. maximum

P2: $z = 3x^2 - 2x\sqrt{y} + y - 8x + 2$

der. 1 řádu

$z'_x = 6x - 2\sqrt{y} - 8 \rightarrow 6x - 2\sqrt{y} - 8 = 0$
$z'_y = -\frac{x}{\sqrt{y}} + 1 \rightarrow -\frac{x}{\sqrt{y}} + 1 = 0 \rightarrow x = \sqrt{y}$

$6x - 2x - 8 = 0$
$4x - 8 = 0$
$x = 2$

SB: $A[2;4]$

der. 2. řádu

	$A[2;4]$
$z''_{xx} = 6$	$C > 0$
$z''_{xy} = -\frac{1}{\sqrt{y}}$	$-\frac{1}{2}$
$z''_{yx} = -\frac{1}{\sqrt{y}}$	$-\frac{1}{2}$
$z''_{yy} = \frac{x}{2\sqrt{y^3}}$	$\frac{1}{8}$

$\Delta_2(A) = \begin{vmatrix} 6 & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{8} \end{vmatrix} = \frac{3}{4} - \frac{1}{4} > 0$
↑ A má extrém
↑ A má lok. minimum

P1: $z = xy + \ln x + y^2 + 3$

$z'_x = y + \frac{1}{x} \rightarrow y + \frac{1}{x} = 0$
$z'_y = x + 2y \rightarrow x + 2y = 0 \Rightarrow x = -2y$

$y - \frac{1}{2y} = 0 \quad /\cdot 2y$
$2y^2 - 1 = 0$
$y^2 = \frac{1}{2} \quad /\sqrt{\;}$
$|y| = \frac{1}{\sqrt{2}} \Rightarrow y_1 = \frac{1}{\sqrt{2}}$
$y_2 = -\frac{1}{\sqrt{2}}$

SB: $A\left[-\sqrt{2}\,;\frac{1}{\sqrt{2}}\right]$
$B\left[\sqrt{2}\,;-\frac{1}{\sqrt{2}}\right]$

der. 2. řádu: $A\left[-\sqrt{2}\,;\frac{1}{\sqrt{2}}\right] \quad B\left[\sqrt{2}\,;-\frac{1}{\sqrt{2}}\right]$

	$A[-\sqrt{2};\frac{1}{\sqrt{2}}]$	$B[\sqrt{2};-\frac{1}{\sqrt{2}}]$
$z''_{xx} = -\frac{1}{y^2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$z''_{xy} = 1$	1	1
$z''_{yx} = 1$	1	1
$z''_{yy} = 2$	2	2

$\Delta_2(A) = \Delta_2(B) = \begin{vmatrix} \frac{1}{2} & 1 \\ 1 & 2 \end{vmatrix} = 1 - 1 = 0$
NEMÁ EXTRÉM
↑ A aj B má sedlový bod

VÁZANÉ EXTRÉMY → $\log y$

$S(x;y) = 0$

P2: $z = x^2 + y^2 - 6x + 4y + 8$ ak $x + y - 3 = 0$

$y = 3 - x \Rightarrow y = 3 - \frac{y}{2} = -\frac{1}{2}$

SB: $A\left[\frac{y}{2}\,;-\frac{1}{2}\right]$

$z = x^2 + (3-x)^2 - 6x + 4(3-x) + 8$
$z = x^2 + 9 - 6x + x^2 - 4x + 12 - 4x + 8$
$z = 2x^2 - 14x + 29$
$z' = 4x - 14 \rightarrow 4x - 14 = 0$
$x = \frac{14}{4} = 3{,}5$
$z'' = 4 > 0$ ∪

↑ A má maximální lokální minimum

P2: $z = x^4 + 2y^2 + 4xy + 3$ ak $x + y = 0$

$x = -y$

$z = x^4 + 2x^2 - 4x^2 + 3$
$z' = 4x^3 - 4x = 4x(x^2 - 1)$
$x_1 = 0 \quad x_2 = 1 \quad x_3 = -1$

SB:

	$A[0;0]$	$B[1;-1]$	$C[-1;1]$
$z'' = 12x^2 - 4$	$-4 < 0$	$8 > 0$	$8 > 0$

↑ A má maximální lok. hod.
↑ B má maximální lok. min.
↑ C má maximální lok. min.