ROVNICA DOTYKOVEJ ROVINY KU GRAFU FCIE. $z=f(x,y)$ V BODE $A[x_0, y_0, z_0]$

$\rho: z - z_0 = z'_x(A)(x-x_0) + z'_y(A)(y-y_0)$

ROVNICA NORMÁLY KU GRAFU FCIE. $z=f(x,y)$ V BODE $A[x_0, y_0, z_0]$ (PARAMETRICKY)

$n: \; x = x_0 + z'_x(A)t$
$\quad y = y_0 + z'_y(A)t$
$\quad z = z_0 - t \qquad ; t \in R$

$f(x)$

$A[x_0, y_0]$

$t: y - y_0 = f'(x_0)(x-x_0)$
$n: y - y_0 = -\frac{1}{f'(x_0)}(x-x_0)$

Pr: $z = 2x^2 - 4y^2 \qquad A[2; 1; ?] = [2; 1; 4]$ (x_0, y_0, z_0)

$? = z_0 = 2 \cdot 2^2 - 4 \cdot 1^2 = 4$

$z'_x = 4x \qquad z'_x(A) = 4 \cdot 2 = 8$

$z'_y = -8y \qquad z'_y(A) = -8 \cdot 1 = -8$

$\rho: z - z_0 = z'_x(A)(x-x_0) + z'_y(A)(y-y_0)$

$z - 4 = 8(x-2) - 8(y-1)$

$z - 4 = 8x - 16 - 8y + 8$

$\boxed{0 = 8x - 8y - 1z - 4}$

$\vec{s}\,(x_0, y_0, z_0)$

$n: x = x_0 + z'_x(A)t = 2 + 8t$
$\quad y = y_0 + z'_y(A)t = 1 - 8t \qquad t \in R$
$\quad z = z_0 - t = 4 - 1t$

Pr: $z = \sqrt{14 - x^2 - y^2} \qquad A[3; 1; ?] = [3; 1; 2]$

$? = z_0 = \sqrt{14 - 3^2 - 1^2} = \sqrt{4} = 2$

$z'_x = \frac{1}{2\sqrt{14-x^2-y^2}}(-2x) \qquad z'_x(A) = \frac{-2 \cdot 3}{2 \cdot \sqrt{14-3^2-1^2}} = \frac{-6}{2 \cdot 2} = -\frac{3}{2}$

$z'_y = \frac{1}{2\sqrt{14-x^2-y^2}}(-2y) \qquad z'_y(A) = \frac{-2 \cdot 1}{2 \cdot \sqrt{14-3^2-1^2}} = \frac{-2}{+2 \cdot 2} = -\frac{1}{2}$

$\rho: z - z_0 = z'_x(A)(x-x_0) + z'_y(A)(y-y_0)$

$z - 2 = -\frac{3}{2}(x-3) - \frac{1}{2}(y-1)$

$z - 2 = -\frac{3}{2}x + \frac{9}{2} - \frac{1}{2}y + \frac{1}{2} \quad / \cdot (2)$

$\boxed{0 = 3x + 1y + 2z - 14}$

$n: x = x_0 + z'_x(A)t = 3 - \frac{3}{2}t$
$\quad y = y_0 + z'_y(A)t = 1 - \frac{1}{2}t \qquad t \in R$
$\quad z = z_0 - t = 2 - t$

$\approx$

$x = 3 + 3s$
$y = 1 + s$
$z = 2 + 2s \qquad s \in R$

$s = -\frac{t}{2}$

Pr: $z = e^{2y} \sin x \qquad A[\frac{\pi}{4}; 0; ?] = [\frac{\pi}{4}; 0; \frac{\sqrt{2}}{2}]$

$? = z_0 = e^0 \cdot \sin\frac{\pi}{4} = 1 \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$

$z'_x = e^{2y}\cos x \qquad z'_x(A) = e^0 \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$z'_y = 2e^{2y}\sin x \qquad z'_y(A) = 2 \cdot e^0 \cdot \sin\frac{\pi}{4} = \sqrt{2}$

$\rho: z - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}(x - \frac{\pi}{4}) + \sqrt{2}(y-0)$

$0 = \sqrt{2}x + 2\sqrt{2}y - 2z + \sqrt{2}(1 - \frac{\pi}{4})$

$n: x = \frac{\pi}{4} - \frac{\sqrt{2}}{2}t, \; y = -\sqrt{2}t, \; z = \frac{\sqrt{2}}{2} - t \qquad t \in R$

alebo $x = \frac{\pi}{4} + \sqrt{2}s, \; y = 2\sqrt{2}s, \; z = \frac{\sqrt{2}}{2} - 2s \qquad s \in R$

Pr: $z = x^2 + (y+3)^2 - 3$

STACIONÁRNY BOD:

der. 1 rádu

$z'_x = 2x \quad \to 2x = 0, \; x = 0$

$z'_y = 2(y+3) \cdot 1 = 2y + 6 \quad \to 2y + 6 = 0, \; y = -3$

$A[0; -3]$

der. 2 rádu

$\boxed{z''_{xx} = 2} > 0$

$z''_{xy} = 0$

$z''_{yx} = 0$

$z''_{yy} = 2$

$\Delta_2(A) = \begin{vmatrix} z''_{xx}(A) & z''_{xy}(A) \\ z''_{yx}(A) & z''_{yy}(A) \end{vmatrix}$

$> 0 \to$ MÁ EXTRÉM
$< 0 \to$ NEMÁ EXTRÉM (SEDLOVÝ BOD)
$= 0$ NEVIEME

$z''_{xx}(A) > 0 \to$ L. MINIMUM

$z''_{xx}(A) < 0 \to$ L. MAXIMUM

$\Delta_2(A) = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 2 \cdot 2 - 0 \cdot 0 = 4 > 0$ MÁ EXTRÉM

v bode A má fcia L. minimum.

Pr: $z = y^2 - x^2 + 3x - 5y + 4$

$z'_x = -2x + 3 \quad \to -2x + 3 = 0, \; x = \frac{3}{2}$

$z'_y = 2y - 5 \quad \to 2y - 5 = 0, \; y = \frac{5}{2}$

SB: $A[\frac{3}{2}; \frac{1}{2}]$

$z''_{xx} = -2$

$z''_{xy} = 0$

$z''_{yx} = 0$

$z''_{yy} = 2$

$\Delta_2(A) = \begin{vmatrix} -2 & 0 \\ 0 & 2 \end{vmatrix} = -4 < 0$

v bode A je sedlový bod

Pr: $z = x^3 + y^3 + 3xy + 2$

1. rád

$z'_x = 3x^2 + 3y$

$z'_y = 3y^2 + 3x$

$3x^2 + 3y = 0 \quad /:3$
$3y^2 + 3x = 0 \quad /:3$

$x^2 + y = 0$
$y^2 + x = 0 \quad \to x = -y^2$

$(-y^2)^2 + y = 0$
$y^4 + y = 0$
$y(y^3 + 1) = 0$
$y = 0 \quad \vee \quad y^3 + 1 = 0$
$\quad y^3 = -1$
$\quad y = -1$

ak $y = 0$ potom $x = 0$
ak $y = -1$ potom $x = -(-1)^2 = -1$

SB: $A[0; 0]$
$\quad B[-1; -1]$

	$A[0;0]$	$B[-1;-1]$
	6.0 =	6.(-1)
$z''_{xx} = 6x$	0	$\boxed{-6} < 0$
$z''_{xy} = 3$	3	3
$z''_{yx} = 3$	3	3
$z''_{yy} = 6y$	0	-6

$\Delta_2(A) = \begin{vmatrix} 0 & 3 \\ 3 & 0 \end{vmatrix} = -9 < 0$

v A je sedlový bod

$\Delta_2(B) = \begin{vmatrix} -6 & 3 \\ 3 & -6 \end{vmatrix} = 36 - 9 = 27 > 0$

v B má extrém

v B má L. maximum

Pr: $z = 24 - x^2 - y^2 + xy + 36x$

$z'_x = -2x + y + 36$

$z'_y = -2y + x$

$-2x + y + 36 = 0$
$-2y + x = 0 \quad /\cdot 2$

$-2x + y = -36$
$2x - 4y = 0 \quad \} +$

$-3y = -36$
$y = 12$

$-2x + y = 0$
$-2x + 12 = 0$
$2x = 12$
$x = 6$

SB: $A[6, 12]$

$z''_{xx} = -2 < 0$

$z''_{xy} = 1$

$z''_{yx} = 1$

$z''_{yy} = -2$

$\Delta_2(A) = \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} = 3 > 0 \to$ MÁ V BODE A EXTRÉM

$z''_{xx}(A) = -2 < 0$

v bode A je L. maximum