(LT^{-1}) INVERZNÁ LAPLACEova TRANSFORMÁCIA

Vieme ak $f(t) \div F(p)$, tak $-t f(t) \div F'(p)$

$\downarrow$ násobenie — derivácia

? $\frac{f(t)}{t} \div$?

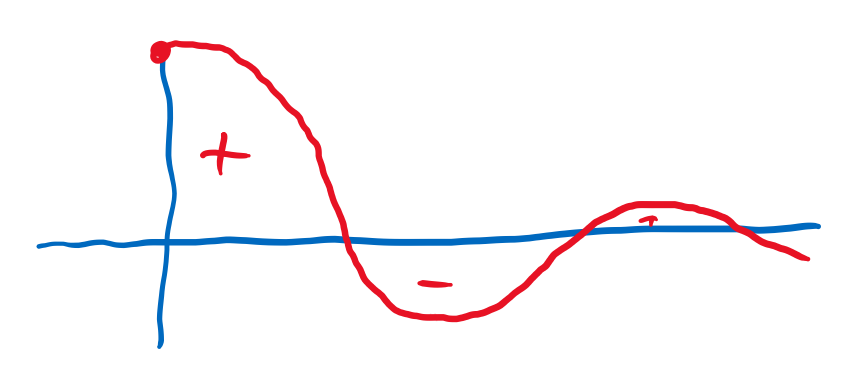
$F(p) \div f(t) = \boxed{\frac{1}{t}}\left(\underbrace{t f(t)}_{-F'(t)}\right)$

$\frac{f(t)}{t}$ ↳ delenie → $\int_p^\infty F(s)\,ds$

ak $f(t) \div F(p)$, tak $\frac{f(t)}{t} \div \int_p^\infty F(s)\,ds$

Pr. $\int_0^\infty \frac{\sin t}{t}\,dt \overset{št.}{=}$ Problém

Prim. fcia nepoznáme



$\mathcal{L}(\sin t) \overset{Tab}{=} \frac{1}{p^2+1} = F(p)$

$\mathcal{L}\left(\frac{\sin t}{t}\right) \overset{Prav}{=} \int_p^\infty \frac{1}{s^2+1}\,ds = \operatorname{arctg} s\Big|_p^\infty = \frac{\pi}{2} - \operatorname{arctg} p$

|| def

$\int_0^\infty \frac{\sin t}{t} e^{-pt}\,dt$ — pre $p=0$ $\boxed{\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}}$

$p=1$ $\int_0^\infty \frac{\sin t}{t} e^{-t}\,dt = \frac{\pi}{2} - \operatorname{arctg} 1 = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

$\int_0^\infty \frac{e^{-at} - e^{-bt}}{t}\,dt$ HRÔZA & DES $b > a > 0$

$\mathcal{L}(e^{-at} - e^{-bt}) \overset{TAB}{=} \frac{1}{p+a} - \frac{1}{p+b}$

$\mathcal{L}\left(\frac{e^{-at}-e^{-bt}}{t}\right) = \int_p^\infty \underbrace{\frac{1}{s+a}}_{\ln(s+a)} - \underbrace{\frac{1}{s+b}}_{\ln(s+b)}\,ds = \ln \frac{s+a}{s+b}\Big|_p^\infty = -\ln \frac{p+a}{p+b}$

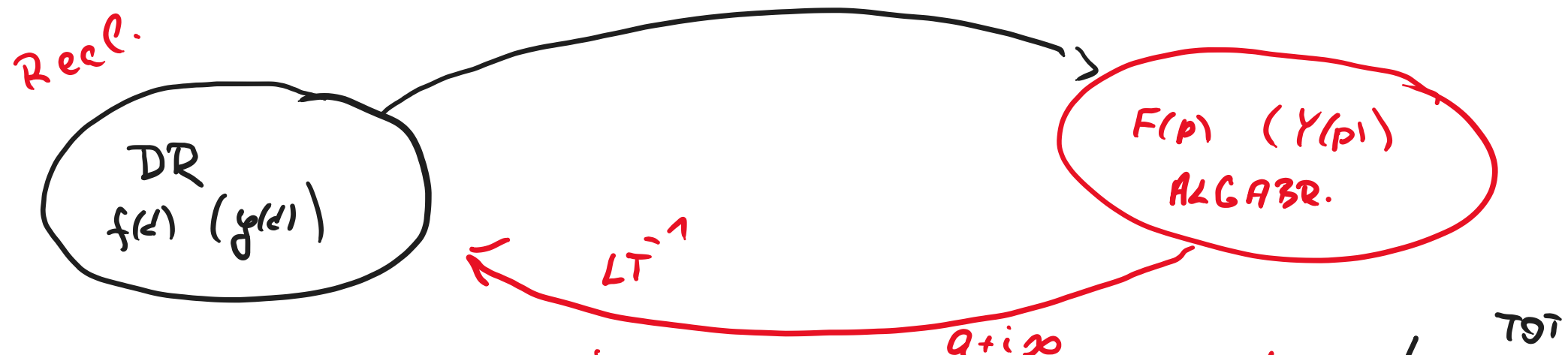
// def

$\int_0^\infty \frac{e^{-at}-e^{-bt}}{t} e^{-pt}\,dt$ — $p=0$ $\int_0^\infty \frac{e^{-at}-e^{-bt}}{t}\,dt = -\ln\frac{a}{b} = \ln\frac{b}{a}$

LT^{-1}

LT $F(p) = \int_0^\infty f(t) e^{-pt}\,dt$ ✓ Kompl.

Reál.



$f(t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} F(p) e^{pt}\,dt$ ✓ TOTO NEBUDEME POUŽÍVAŤ

1. spôsob LT^{-1} návrat z Kompl.→R.

$1 \rightleftarrows \frac{1}{p}$ ✓ $t^n \rightleftarrows \frac{n!}{p^{n+1}}$ ✓

$e^{at} \rightleftarrows \frac{1}{p-a}$ ✓ $\sin \omega t \rightleftarrows \frac{\omega}{p^2+\omega^2}$ ✓

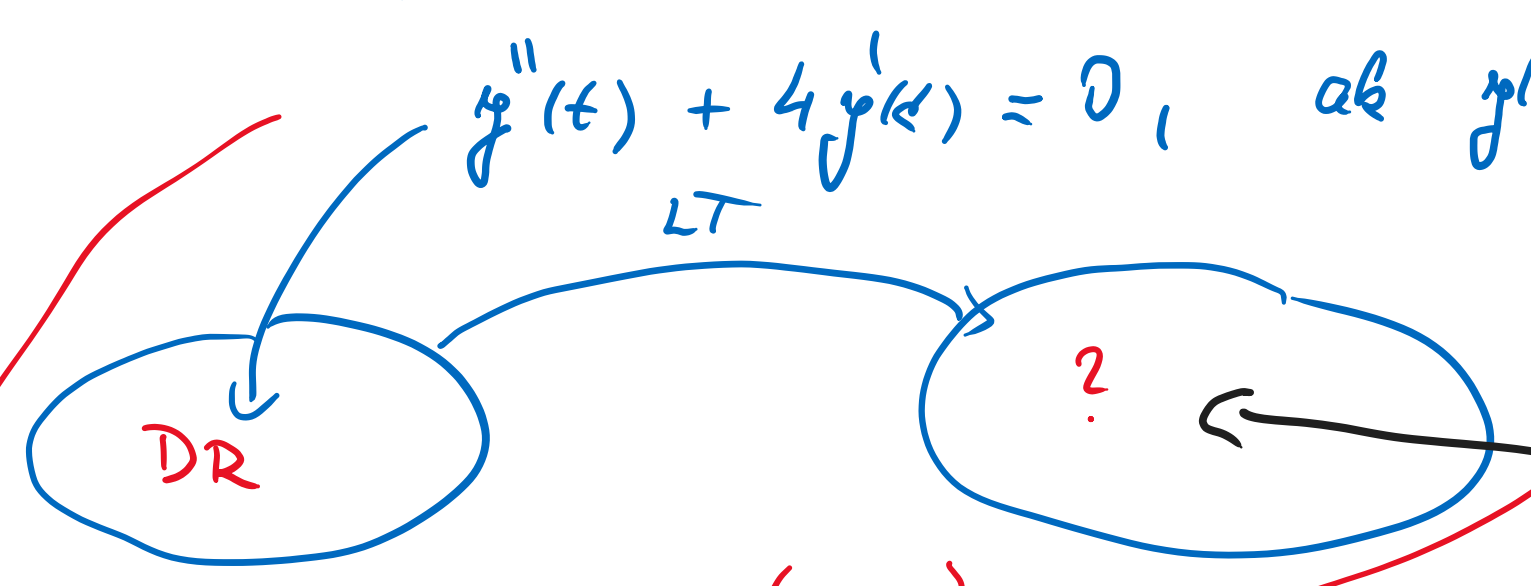
2. spôsob LT^{-1} cez rezíduá

ak $a_1, a_2, \ldots, a_k$ sú póly ([illegible]) funkcie $F(p)$, tak

$f(t) = \sum_{i=1}^k \operatorname{res}\left[\underbrace{F(p)e^{pt}}\right]_{p=a_i}$

P1) Riešte pomocou LT

$y''(t) + 4y'(t) = 0$, ak $y(0) = 1$, $y'(0) = -6$



$\mathcal{L}(y(t)) = Y(p)$ $(F(p))$ $y(t) \div Y(p)$

$\mathcal{L}(y'(t)) \overset{ŤAHÁK}{=} p\cdot Y(p) - y(0) = pY(p) - 1$ ←

$\mathcal{L}(y''(t)) \overset{ŤAH}{=} p^2Y(p) - py(0) - y'(0) = p^2Y(p) - p + 6$

$\boxed{p^2Y(p) - p + 6 + 4pY(p) - 4 = 0}$

DR vyriešené v Komp. obore TREBA SA VRÁTIŤ ? $y(t)$?

$Y(p) = \frac{p-2}{p^2+4p}$

Pozrieme do ŤAHÁKA a NIE ZJEDNODUŠ. PAR. ZL.

$Y(p) = \frac{p-2}{p^2+4p} = \frac{p-2}{p(p+4)} = \frac{A}{p} + \frac{B}{p+4}$ $A = \frac{-2}{4} = -\frac{1}{2}$ $B = \frac{-6}{-4} = \frac{3}{2}$

$Y(p) = -\frac{1}{2}\,\frac{1}{p} + \frac{3}{2}\,\frac{1}{p+4}$

TAB

$y(t) = -\frac{1}{2}\cdot 1 + \frac{3}{2}\cdot e^{-4t} = -\frac{1}{2} + \frac{3e^{-4t}}{2}$

2. spôsob cez rezíduá

$Y(p) = \frac{p-2}{p(p+4)}$ póly 0 (1. rád), −4 (1. rád)

$y(t) = \operatorname{res} \frac{p-2}{p(p+4)} e^{pt}\Big|_0 + \operatorname{res} \frac{p-2}{p\,\cancel{(p+4)}} e^{pt}\Big|_{-4}$

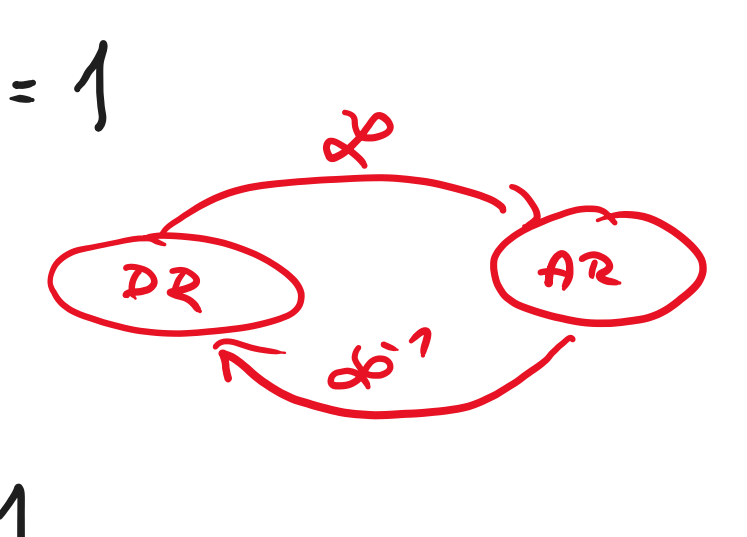
$\lim_{p\to 0} \frac{p-2}{p+4} e^{pt} = -\frac{2}{4}$ $+$ $\frac{-6}{-4} e^{-pt}$

$y(t)$

Riešte DR

$y''(t) - y(t) = t$; $y(0) = 1$, $y'(0) = 1$

$\mathcal{L}(y''(t) - y(t)) = \mathcal{L}(t)$



ŤAHÁK

$\mathcal{L}(y(t)) = Y(p)$

$\mathcal{L}(y''(t)) \overset{T}{=} p^2Y(p) - py(0) - y'(0) = p^2Y(p) - p - 1$

$\mathcal{L}(t) = \frac{1}{p^2}$

1 (-) 10

$p^2Y(p) - p - 1 - Y(p) = \frac{1}{p^2}$ ①

$Y(p) = \frac{\frac{1}{p^2} + p + 1}{p^2-1} = \boxed{\frac{p^3+p^2+1}{p^2(p^2-1)}}$

1Q -> 2

$Y(p) = \frac{p^3+p^2+1}{p^2(p-1)(p+1)} = \frac{A}{p} + \frac{B}{p^2} + \frac{C}{p-1} + \frac{D}{p+1}$ $B = -1$, $C = \frac{3}{2}$, $D = \frac{-1}{-2} = \frac{1}{2}$ $p=-1$ $p=1$

A: p^3: $1 = A + C + D$

IQ 8

$p^3+p^2+1 = Ap(p-1)(p+1) + B(p-1)(p+1) + Cp^2(p+1) + Dp^2(p-1)$

$y(t) = A\cdot 1 + Bt + Ce^{t} + De^{-t}$

1.Q 1

2. spôs. $Y(p)$ $\frac{p^3+p^2+1}{p^2(p-1)(p+1)} e^{pt} = g(p) = Y(p)e^{pt}$

0 dvoj.
1 prv.
−1 prv.

$y(t) = \operatorname{res} g(0) + \operatorname{res} g(1) + \operatorname{res} g(-1)$

$\operatorname{res} g(1) = \frac{3}{2}e^t$ ✓ $\operatorname{res} g(-1) = \frac{-1}{-2}e^{-t}$ ✓

$\operatorname{res} g(0) = \lim_{p\to 0}\left(\boxed{\frac{p^3+p^2+1}{p^2-1}}e^{pt}\right)' = \lim_{p\to 0}\left[\frac{(3p^2+2p)(p^2-1) - (p^3+p^2+1)2p}{(p^2-1)^2}e^{pt} + \frac{p^3+p^2+1}{p^2-1}te^{pt}\right]$ ✓

$x''(t) - 2x'(t) + x(t) = e^t$; $x(0) = 0$, $x'(0) = 1$

$\mathcal{L}(x(t)) = X(p)$ $L(x'(t)) = pX(p) - 0$ $L(x''(t)) = p^2X(p) - 1$ $L(e^t) = \frac{1}{p-1}$

$p^2X(p) - 1 - 2pX + X = \frac{1}{p-1}$

$X(\underbrace{p^2 - 2p + 1}_{(p-1)^2}) = \frac{1}{p-1} + 1 = \frac{1+p-1}{p-1} = \frac{p}{p-1}$

$X(p) = \frac{p}{(p-1)^3}$ 1 − pól 3. rádu

$g(p) = X(p)e^{pt} = \frac{p}{(p-1)^3}e^{pt}$ $' = \frac{d}{dp}$

$x(t) = \operatorname{res} g(1) = \frac{1}{2}\lim_{p\to 1}\left(pe^{pt}\right)'' = \frac{1}{2}\lim_{p\to 1}\left(e^{pt} + pte^{pt}\right)'$

$= \frac{1}{2}\lim_{p\to 1}\left[te^{pt} + te^{pt} + pt^2e^{pt}\right] = \frac{1}{2}\left(2te^t + t^2e^t\right)$

$\frac{p}{(p-1)^3} = \frac{A}{p-1} + \frac{B}{(p-1)^2} + \frac{C}{(p-1)^3}$ $C = 1$

$\boxed{\begin{aligned}\frac{1}{p^2} &\longleftrightarrow t\\ \frac{1}{(p-1)^2} &\longleftrightarrow te^{pt}\end{aligned}}$

$p = A(p-1)^2 + B(p-1) + C$ ← MÁM ROZKLAD ?!?

$\frac{p\overbrace{-1+1}^{0}}{(p-1)^3} = \frac{1}{(p-1)^2} + \frac{1}{(p-1)^3}$