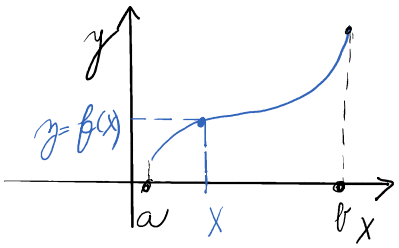
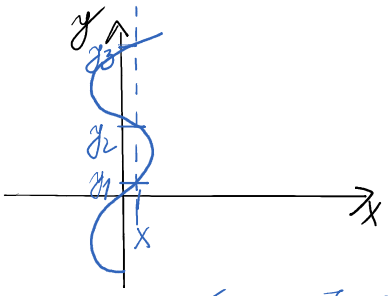
Pojem funkcie

streda 23. októbra 2024 7:30



$\forall x \; \exists$ najviac 1 y

je graf funkcie



pre dané x $\exists$ 3 hodnoty y

nie je graf funkcie

$y = f(x)$

y ... závislá premenná

x ... nezávislá premenná

$D(f)$ ... definičný obor

$H(f)$ ... obor hodnôt

PRÍKLADY

1) $y = x^2 - 4x$ $\quad D(f) = \mathbb{R}$

2) $y = \frac{x-2}{x^2-9}$ $\quad D(f): x^2 - 9 \neq 0 \quad x \neq \pm 3 \quad D(f) = \mathbb{R} - \{\pm 3\}$

3) $y = \log_2 x$ $\quad D(f): x > 0 \quad D(f) = \mathbb{R}^+ = (0, \infty)$

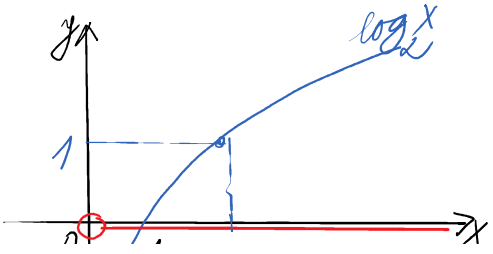
4) $y = \sqrt{x-4}$ $\quad D(f): x - 4 \geq 0 \quad x \geq 4 \quad D(f) = \langle 4, \infty)$

PRAVIDLÁ NA URČOVANIE $D(f)$:

1. VÝRAZ V MENOVATELI RÔZNY OD NULY ($\neq 0$)
2. VÝRAZ POD LOGARITMOM KLADNÝ (> 0)
3. VÝRAZ POD PÁRNOU ODMOCNINOU NEZÁPORNÝ ($\geq$)

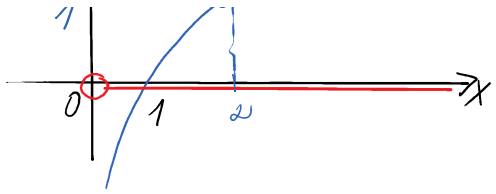
GRAF FUNKCIE

$y = \log_2 x$



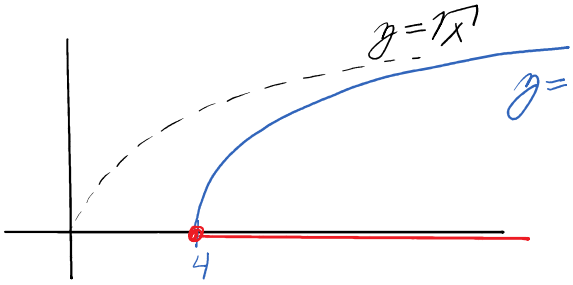
$D(f) = (0, \infty)$

$H(f) = \mathbb{R}$



$H(f) = \mathbb{R}$

$y = \sqrt{x-4}$

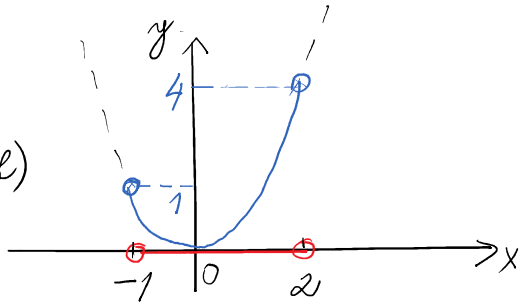


$D(f) = \langle 4, \infty)$

$H(f) = \langle 0, \infty)$

OHRANIČENÁ FUNKCIA

1) $y = x^2$ $\quad D(f) = \mathbb{R}$



NECH $S = (-1, 2) \subset D(f)$

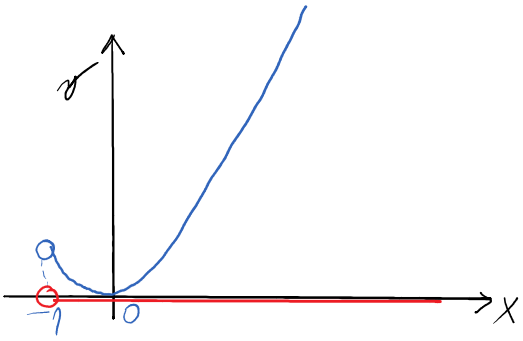
$f(S) = \langle 0, 4)$

ZDOLA OHRANIČENÁ $\quad k = -5 \quad \forall x \in S \quad f(x) \geq -5$

ZHORA OHRANIČENÁ $\quad K = 10 \quad \forall x \in S \quad f(x) \leq 10$

$\Rightarrow$ FUNKCIA JE NA MNOŽINE S OHRANIČENÁ

2) $y = x^2$



NECH $S = (-1, \infty)$

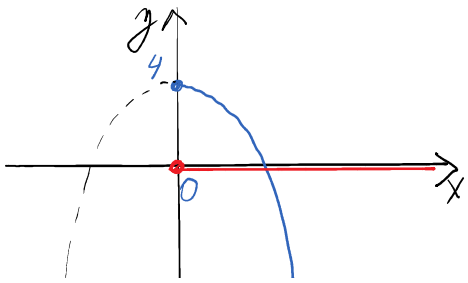
$f(S) = \langle 0, \infty)$

ZDOLA OHRANIČENÁ

ZHORA NEOHRANIČENÁ

$\Rightarrow$ FUNKCIA NIE JE NA MNOŽINE S OHRANIČENÁ

3) $y = 4 - x^2$



NECH $S = \langle 0, \infty)$

$f(S) = (-\infty, 4\rangle$

ZDOLA NEOHRANIČENÁ

ZHORA OHRANIČENÁ

$\Rightarrow$ FUNKCIA NIE JE NA MNOŽINE S OHRANIČENÁ

EXTRÉMY FUNKCIE : MAXIMUM / MINIMUM

1) $f(x) = x^2 \quad S = (-1,2)$

$\min_{(-1,2)} x^2 = 0 \qquad \max_{(-1,2)} x^2 \quad \nexists$

2) $f(x) = x^2 \quad S = (-1,\infty)$

$\min_{(-1,\infty)} x^2 = 0 \qquad \max_{(-1,\infty)} x^2 \quad \nexists$

3) $f(x) = 4 - x^2 \quad S = (0,\infty)$

$\min_{(0,\infty)} (4-x^2) \quad \nexists \qquad \max_{(0,\infty)} (4-x^2) = 4$

MONOTÓNNOSŤ FUNKCIE RASTÚCOSŤ / KLESAJÚCOSŤ

$y = x^2 \quad S = (0,\infty)$

$\forall x_1, x_2 \in (0,\infty)$:

$x_1 < x_2 \quad f(x_1) < f(x_2)$

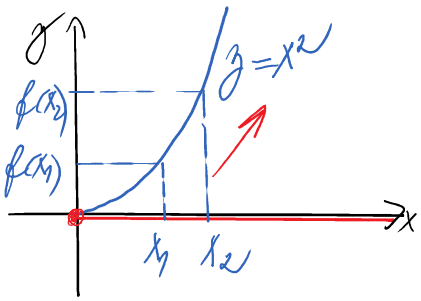
RASTÚCA NA $(0,\infty)$

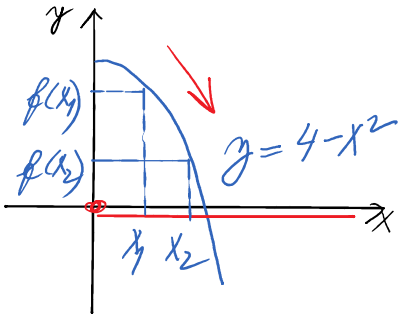
$y = 4 - x^2 \quad S = (0,\infty)$

$\forall x_1, x_2 \in (0,\infty)$

$x_1 < x_2 \quad f(x_1) > f(x_2)$

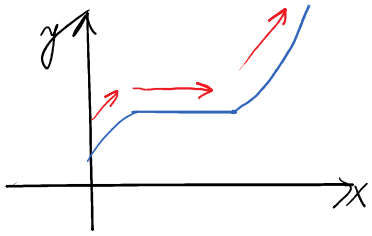
KLESAJÚCA NA $(0,\infty)$





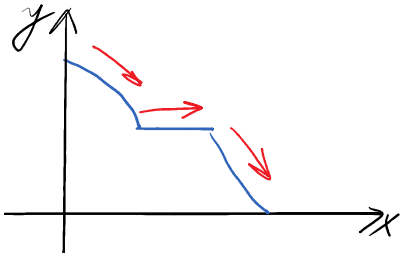
NEKLESAJÚCA

$x_1 < x_2 \quad f(x_1) \leq f(x_2)$

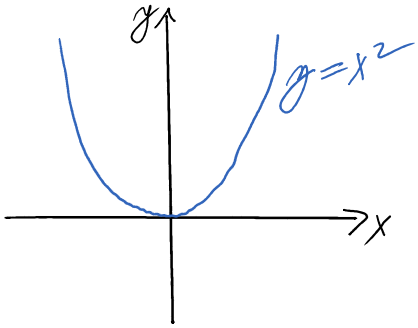


NERASTÚCA

$x_1 < x_2 \quad f(x_1) \geq f(x_2)$



PÁRNOSŤ / NEPÁRNOSŤ



$f(x) = x^2 \qquad D(f) = \mathbb{R}$

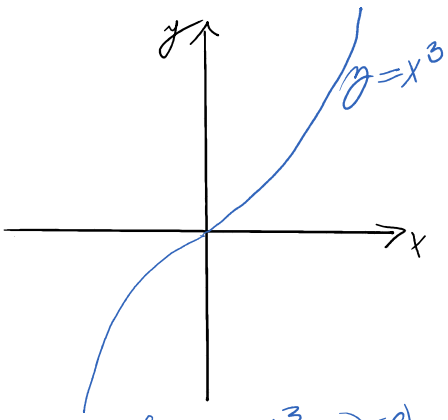
$\forall x \in D(f) \quad -x \in D(f)$ a platí:

$f(-x) = f(x)$

$(-x)^2 = x^2$

$x^2 = x^2$

PÁRNA



$f(x) = x^3 \qquad D(f) = \mathbb{R}$

$\forall x \in D(f) \quad -x \in D(f)$ a platí

$f(-x) = -f(x)$

$(-x)^3 = -(x^3)$

$-x^3 = -x^3$

NEPÁRNA

$f(x) = \cos x \qquad D(f) = \mathbb{R}$

$\cos(-x) = \cos x$ P

$f(x) = \sin x \qquad D(f) = \mathbb{R}$

$\sin(-x) = -\sin x$ NP

PERIODICKOSŤ FUNKCIE

$\forall x \in D(f) \quad x \pm p \in D(f) : \quad f(x \pm p) = f(x)$

p ↑ PERIÓDA

$\cos(x + 2\pi) = \cos x \qquad p = 2\pi$

$\sin(x + 2\pi) = \sin x \qquad p = 2\pi$

$\operatorname{tg}(x + \pi) = \operatorname{tg} x \qquad p = \pi$

$\operatorname{cotg}(x + \pi) = \operatorname{cotg} x \qquad p = \pi$